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Inventory Management Modeling With Markov Decision Process (Mdp) For Equitable Distribution Of Supplies Under Uncertainty

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INVENTORY MANAGEMENT MODELING WITH MARKOV DECISION PROCESS (MDP) FOR EQUITABLE DISTRIBUTION OF SUPPLIES UNDER UNCERTAINTY

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Sefakor Fianu was born on August 20, 1987, in Ghana. She received her Bachelor of Science Degree in Chemical Engineering from Kwame Nkrumah University of Science and Technology (Ghana) in 2010. She has been a Master of Science candidate for Industrial & Systems Engineering since August, 2013 until May, 2015.

Dedication

This thesis work is dedicated to my husband whose countless support and encouragement has brought me this far in graduate education and life. Thank you for all the weekends and sleepless nights you helped me so that my work would go a little quicker.

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I am extremely grateful to the Almighty God for the successful completion of my Master’s Degree program. My sincere gratitude goes to my academic supervisor Dr. Lauren Davis for her financial support, leadership and direction over the years. Her comments, suggestions and corrections were immeasurably helpful in stirring this research forward.

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Abstract

Food insecurity is defined as the situation where people are not able to access enough food at all times for an active, healthy life. The 2012 food security report stated that 49 million Americans including children lived in food insecure households. Many individuals suffering from food insecurity obtain assistance from governmental programs and nonprofit agencies. Food banks are one of many non-profit organizations assisting in the fight against hunger. They serve communities by distributing food to those in need through charitable agencies. Many of the food distributed by the food bank comes from donations. These donations are received from various sources in uncertain quantities at random points in time. Working with this type of uncertainty in supplies can be very challenging. This thesis aims at developing a decision-making model that will assist food banks to distribute supplies equitably as well as measure their performance using the pounds per person in poverty indicator. This model will also assist them in managing their inventory levels in order to meet the demand of aid recipients with the random supplies (donations) to the food bank.

CHAPTER 1

Introduction

1.1 Food Insecurity

Food insecurity is defined by United States Department of Agricultural, Economic Research Service (USDA, ERS) as the situation where people are not able to access enough food at all times for an active, healthy life. The United Nations Food and Agriculture Organization (UNFAO), estimates that nearly 870 million people out of the 7.1 billion people in the world suffered from chronic malnutrition in the years 2010-2012 (FAO et al., 2012). In the United States, 49 million Americans including children lived in food insecure households in 2012 (Coleman-Jensen et al., 2013). Children are mostly shielded from the disrupted eating patterns and reduced food intake that characterize food insecure households. However, the 2012 report on household food security stated that 3.9 million households were unable to provide enough nutritious food for their children (Coleman-Jensen et al., 2013). Figure 1 shows the percentage of households that were food insecure in 2012.

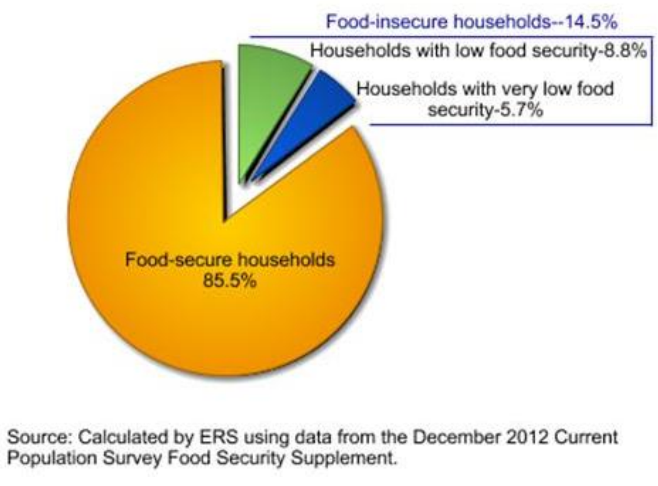

Figure 1. U.S. households by food security status, 2012.

Since 1995, the U.S. Department of Agriculture has collected data annually on food access and adequacy, food spending, and sources of food assistance for the U.S. population. A major motivation for this data collection is to provide information about the prevalence and severity of food insecurity in U.S. households. Figure 2 shows an increasing trend in the percentage of households that were food insecure from 1995 to 2012 (Coleman-Jensen et al., 2013).

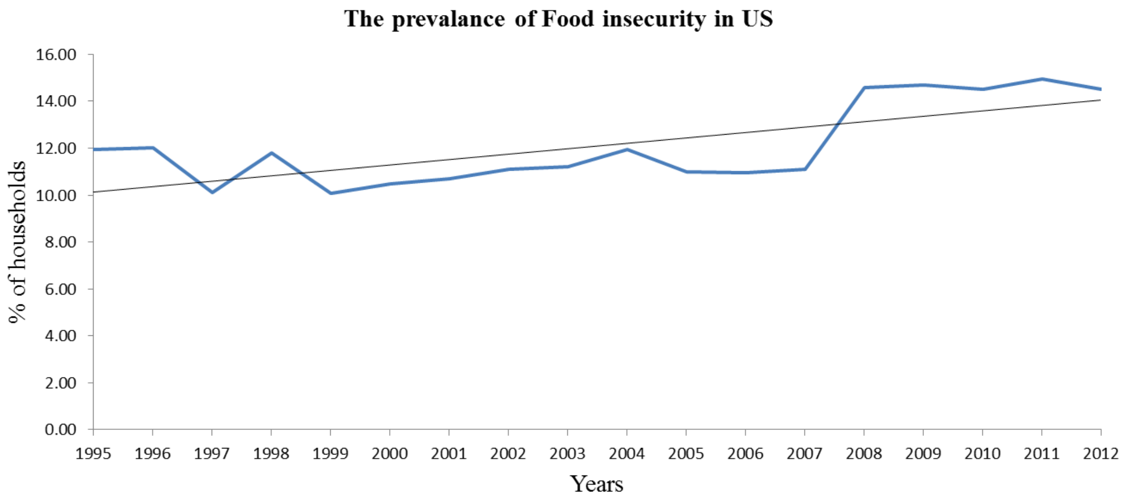

Figure 2. U.S. The prevalence of food insecurity in the United States.

(Source: Calculated by ERS based on Current Population Survey Food Security)

The prevalence rates of food insecurity can also be measured at the state level and it varies significantly from one state to the other. Figure 3 illustrates ten states that exhibited significantly higher household food insecurity rates than the national average of 14.7% from the year 2000 - 2012.

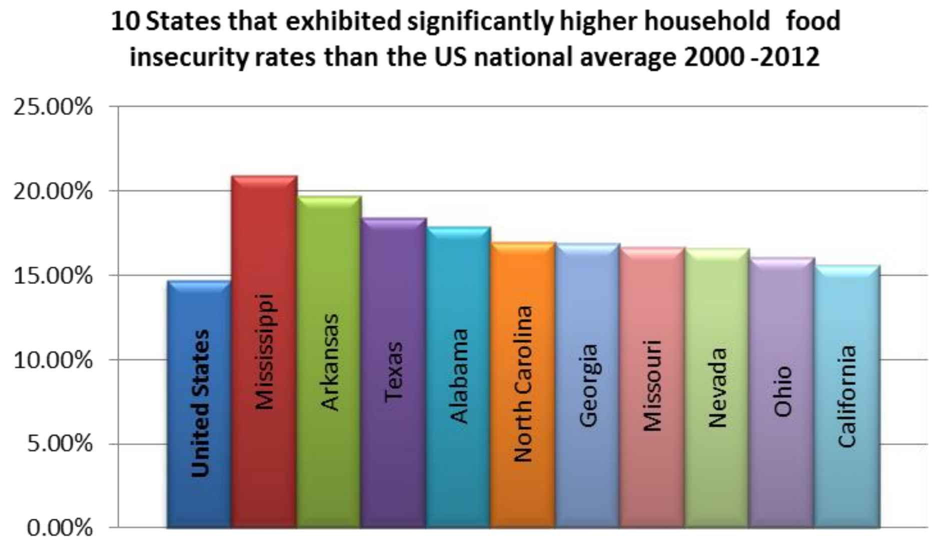

Figure 3. Top 10 states that exhibited higher household food insecurity rates than the U.S. national average 2000-2012.

1.2 Causes of Food Insecurity

Food insecurity may be caused by natural or man-made disaster such as drought, war or earthquakes. It can also be caused by persistent poverty (Barrett, 2010). People may be starving not because of scarcity of food but due to financial difficulty to cover the cost of three square meals a day. In some cases, people have to make a tradeoff between using the money to cover medical expenses or other bills and buying food. In 2012, 46.5 million people (15.0 %) in the United States were in poverty (DeNavas-Walt et al., 2013); this is close to the 49 million people that were reported to be food insecure in the same year (Coleman-Jensen et al., 2013). This shows that there is a correlation between poverty and food insecurity.

The United States (U.S.) government has established several public assistance programs to address the problem of food insecurity. The U.S. Department of Agriculture (USDA) food and nutrition assistance program provides safe, sufficient and nutritious food supply for people at risk of hunger. The Supplemental Nutrition Assistance Program (SNAP) originally established as the

Food Stamp Program ensures access to healthy food for low-income households. Women, Infants, and Children (WIC) program helps ensure the nutritional health of pregnant, postpartum and breastfeeding mothers, infants and children. The Emergency Food Assistance Program (TEFAP) provides food and administrative funds to states to supplement the diets of low-income individuals, including the elderly.

1.3 Feeding America and its Network of Food Banks

Feeding America formally known as America’s second harvest is the nation's leading domestic hunger-relief organization whose mission is to end hunger by feeding America's hungry through a nationwide network of about 200 member food banks and distribution centers across the country (FeedingAmerica, 2014). Feeding America is the parent hunger-relief organization that gave birth to all the local food banks; as such, they provide administrative support, training of personnel, standards for food safety and standards for food distribution to the food banks. They also facilitate the receipt of food and funds from the Federal government through programs such as TEFAP and distribute to local food banks based on requests through a bidding system.

The local food banks, though under the umbrella of Feeding America, remain largely independent with their own management system and budget. They solicit for funds, food and supplies from individuals, groups, farmers, local manufacturers and retailers. Donors such as large companies may decide to donate to food banks as part of their corporate responsibility program. In addition individual donors may also decide to donate in response to solicitation for supplies by the food bank. The types of items donated are based on the following factors: (i) whether they are purchased specifically for donations per what the food bank wanted; (ii) what the donor felt like is appropriate to donate; or (iii) they are surplus from existing supplies. These items received represent sources of supply that enable them to meet the demand of the people at risk of hunger. The food banks do not have any control on the types of food, the quantity donated, and frequency of donations. As a result, they mostly do not have the amount and type of supplies they actually need to enable them to meet the needs of their clients (people at risk of hunger) in a timely manner.

1.4 Performance Measurement

Neely et al. (2002) define performance measurement as "the process of quantifying the efficiency and effectiveness of past actions". Organizations periodically evaluate their performance over a period of time as a way of being accountable to their stakeholders and also to see how they are performing. This enables them to identify the deficiencies in their systems and find ways of improving their systems. There are several challenges that exist when measuring the performance of hunger-relief organizations. Some of these challenges include the immateriality of their services, immeasurability of their missions, unknowable outcomes, and varied interests of stakeholders (Balcik and Beamon, 2008). Despite these challenges hunger-relief organizations need to measure their performance for the following reasons. Firstly, to evaluate their previous performance and improve on their ability to meet demand in subsequent times. Secondly to be accountable to their benefactors, beneficiaries, staff, volunteers, the media, and the public in general. Thirdly to be able to compete with a burgeoning number of agencies, for scarce donor funding (Kaplan, 2001).

1.4.1 Pounds per person in poverty (PPIP)

In view of the need to be able to measure the performance of hunger-relief organizations such as food banks, Feeding America has proposed a performance indicator to members in its network which is the pounds distributed per person in poverty (PPIP) by service area. The PPIP indicator does not directly measure the number of pounds of food distributed to the people in poverty served by a given food bank, but rather divides the total number of pounds distributed by a given member food bank by the entire poverty population in the member's service area (Gillis, 2010). Feeding America's benchmark is to distribute at least 75 pounds of products for each person in poverty (over a 12 month period). It's over a 12-month period because the 12 months represent one fiscal year for the food banks; hence Feeding America would like to know how much food its members distribute over an entire fiscal year. A food bank is successful if its PPIP is 75 or more. Thus, service areas that have a PPIP below 75 are considered to be under served.

1.4.2 Food bank of central and eastern North Carolina

The Food bank of Central and Eastern North Carolina (FBCENC), a member of Feeding America network, has been providing food to people at risk of hunger in 34 counties in Central and Eastern North Carolina for over 30 years. The FBCENC comprises six branches located in the Wilmington, Durham, Raleigh, Sandhills, Greenville, and New Bern areas in North Carolina. In addition, these branches serve as warehouses for the food bank. In fiscal year 2011-12 (FY1112), the food bank distributed nearly 45 million pounds of food and non-food essentials through its partner agencies. The partner agencies consist of emergency food programs such as soup kitchens, food pantries, kid's café, homeless shelters, elderly nutrition programs and recognized churches. These partner agencies also serve more than 500,000 individuals at risk of hunger across the 34 counties. The FBCENC receives donations from State and Federal government, individuals, organizations, manufacturers and retailers.

1.5 Problem Statement and Motivation

More than 70% of the food received by the FBCENC is from donations, which are completely voluntary. This implies that the supplies to the food bank are based on the goodwill of donors and is therefore subject to significant variations due to the fact that donors are not obligated to give any particular type and quantity of supplies. Consequently, donations may not be frequent, may be almost expiring and may not be suitable for consumption after a few days, or the items are not what is actually being demanded.

Working with this type of uncertainty in supplies can be very challenging. Nevertheless, management at the food bank needs to be able to adequately plan how to ensure that there are supplies coming in despite all these factors and also to plan the distribution of supplies to ensure food shortages are avoided. In order to properly manage the distribution of donations, the FBCENC initiated the fair share program, which uses readily available poverty rates in each county to provide a blueprint of the areas in greatest need of food and other supplies. This information is used by the FBCENC's Operations Team to move donated food and other product through its six warehouses to the partner agencies in the counties and to the people at risk of hunger. The management at FBCENC wants to use the performance indicator (PPIP) proposed by Feeding America to measure their performance. This thesis aims at developing a decision-making model that will assist the FBCENC to distribute supplies equitably as well as measure their performance using the pounds per person in poverty indicator. And also assist them to manage their inventory levels in order to meet the demand of aid recipients with the random supplies (donations) to the food bank.

1.6 Research Objectives

The main objective of this project is to provide a platform (decision making model) for the food bank's Decision Makers specifically, to achieve the following goals:

- Find an optimal distribution policy that maximizes equity in the distribution of supplies using the PPIP;

- Determine reward associated with the optimal distribution policy;
- Estimate the number of counties whose PPIPs fall below the 75;
- Estimate the amount of unsatisfied demand for the counties whose PPIP are below 75.

1.7 Organization of Thesis

The remainder of the thesis is outlined as follows. Chapter 2 summarizes the related literature. Chapter 3 outlines the methodology employed. Chapter 4 evaluates the methodology using data from FBCENC and describes the experimental design constructed to answer the research questions. The results and key insights from the model are summarized in Chapter 5. Chapter 6 contains concluding remarks and extensions for future work.

CHAPTER 2

Literature Review

2.1 Introduction

This chapter is divided into three sections. The first section is a review of humanitarian relief operations and the various types of humanitarian relief. Relief inventories and their challenges are discussed. The main challenges identified are supply uncertainty, demand uncertainty and equitable allocation of supplies. The second section describes decision-making and highlights decision-making models to solve inventory control problems. The third section describes the use of Markov decision-making models to solve inventory control problems.

2.2 A Review of Humanitarian Relief Operations

Humanitarian relief is assistance in the form of food, water, medicine, shelter and supplies provided to people affected by emergencies (Balcik and Beamon, 2008). Emergencies could vary from food insecurity primarily caused by economic hardships to large-scale emergencies caused by natural or man-made disasters such as war, earthquakes or floods (Mohan et al., 2013). Humanitarian relief operations often include preparation, planning, assessment, appeal, mobilization, procurement, warehousing, transportation and distribution of goods and services to the affected people (Blanco and Goentzel, 2006). Celik et al. (2012) divide humanitarian relief into two main categories: disaster-related operations and long-term humanitarian development related operations. The former concentrates on providing basic necessities and services to relieve the suffering and save the lives of the vulnerable in the interim. However the latter focuses on the long–term eradication of the root causes of vulnerability through capacity building. This is done by special interventions programs such as the transfer of knowledge and resources through mentoring, workshops, trainings and infrastructure development.

From the above classification of humanitarian relief this research falls into the category of long-term humanitarian development related operations. Though the research focuses on the long-term relief offered to people at risk of hunger, disaster related relief operations will also be reviewed due to its similarities with our study. The similarities are; supply chain structure, which allows the flow of material from donors to beneficiaries and objective functions, which are to alleviate human suffering and to distribute resources equitably. However, disaster related issues have different constraints compared to our studies. These constraints include the sudden occurrence of demand, extreme urgency and chaotic environments.

Humanitarian Supply Chain (HSC) or Humanitarian Logistics (HL) is a network of organizations that ensure the effective and efficient solicitation, transportation, warehousing and the distribution of supplies and the provision of other services to people in need. Due to the increasing trends of natural disasters and food insecurity coupled with the need for accountability, relief agencies should be able to manage their supply chains effectively to improve their responsiveness and efficiency. In view of these reasons, HSC management has attracted significant attention (Altay and Green Iii, 2006; Blanco and Goentzel, 2006; Balcik et al., 2010; Galindo and Batta, 2013). The areas that have attracted the most research can be grouped into three main categories; transportation, facility location and inventory management, as shown in Table 1.

Research in transportation highlights vehicle routing problems that determine effective distribution of relief items and the modes of transportation (such as helicopters) to reach areas that are not accessible by road. Research on facility locations emphasizes the pre-positioning of supplies in the pre-disaster phase and strategic positioning of distribution centers close to demand points. Research on inventory management focuses on determining the item quantities required at various distribution centers, how to distribute supplies equitably, order frequency, and the appropriate amount of safety stock to have in order to prevent supply interruptions.

Table 1

Work done by various researchers in the three main areas of HSC

Transportation	**Facility location**	**Inventory control**
(Davis et al., 2014)	(Roh et al., 2013)	(Das and Hanaoka, 2014)
(Liberatore et al., 2014)	(Duran et al., 2011)	(Davis et al., 2013)
(Nikbakhsh and Zanjirani Farahani, 2011)	(Rawls and Turnquist, 2010)	(Rawls and Turnquist, 2012)
(Jr and Taskin, 2008)	(Balcik and Beamon, 2008)	(Qin et al., 2012)
(Campbell et al., 2008)	(Ukkusuri and Yushimito, 2008)	(Rottkemper et al., 2012)
(Jahre et al., 2007)	(Jia et al., 2007)	(Bozorgi-Amiri et al., 2013)
(Barbarosoğlu and Arda, 2004)	(Tzeng et al., 2007)	(Chang et al., 2007)
(Sakakibara et al., 2004)	(Yi and Özdamar, 2007)	(Ozbay and Ozguven, 2007)
(Barbarosoğlu et al., 2002)		

2.2.1 Relief inventories

Relief inventories are referred to as social inventories because they serve broad social objectives as opposed to being used for the benefit of an individual enterprise (Whybark, 2007). These inventories are unique in terms of their source of supplies, objectives, recipients of services, workers, performance measurement and the level of uncertainty and risk they have to deal with (Van Wassenhove, 2005; Balcik and Beamon, 2008). Given the importance of disaster relief operations, the amount of research available on relief inventories is little compared to research on commercial inventories. Commercial inventories usually deal with predetermined suppliers, predetermined facility location sites, and predictable demand; all of these factors are unknown in relief inventories (Cassidy, 2003). In terms of objectives, commercial inventories aim at increasing profits whereas relief inventories aim at alleviating the suffering of vulnerable people (Anisya and Kopczak, 2005). These differences between relief inventories and commercial inventories (Balcik and Beamon, 2008; Balcik et al., 2010) prevent the application of commercial inventory models directly to relief inventories.

2.2.2 Challenges of relief inventories

In contrast to managing commercial inventories, supply is highly uncertain because it is dependent on donations that are constantly evolving. This creates a major challenge in relief inventory management since without supply there will be no distribution. The defining source of revenue for hunger-relief organizations is scarce government funding and irregular charitable donations from individuals and corporations. For disaster relief operations, a preliminary appeal for donations of cash and relief supplies is often made within 36 hours of the onset of a disaster (Anisya, 2003). Fundraising and sale of goods and services is also another means by which relief organizations mobilize resources for their activities.

Issues with supply uncertainty range from ability of a donor to give supplies, the varied quantities of supplies donated and the receipt of unsolicited and sometimes unwanted donations (Chomilier et al., 2003). Most often than not, relief agencies have to deal with food and medications that are highly perishable or past their expiry dates. They have to sort and review the items for quality before distribution to end users. These unwanted items clog their warehouses or distribution centers and thus increase the inventory cost of handling and holding these items (Sowinski, 2003; Murray, 2005). As a result, most relief operations have incinerators to help destroy these unwanted items (Murray, 2005).

Concerns have been raised about the nutritional quality of emergency foods due to the increasing rates of obesity among the food insecure (Campbell et al., 2013; Ross et al., 2013). Hunger-relief organizations have been asked to implement nutritional policies that will help improve the nutritional quality of the food they provide to aid recipients (Handforth et al., 2013; Shimada et al., 2013; Webb, 2013). Studies done by Campbell et al. (2013) show that just a handful of hunger-relief organizations are giving out more fresh produce and have programs where clients can choose what food items they want rather than being handed a bag filled with random groceries consisting of mostly canned foods. Most hunger-relief organizations express difficulty in implementing the nutritional policies as these have the potential of reducing the total amount of food that is donated, discomfort in choosing which foods should not be permitted, and concerns about jeopardizing relationships with donors and partners (Handforth et al., 2013). Hoisington et al. (2011) proposed “My Pyramid Day analysis tool” to help relief inventory managers to monitor key food items that are not donated. They can then conduct targeted food drives requesting donations of those nutritious foods or they can purchase these foods from available funds for distribution.

On the demand side uncertainties arise with quantifying the needs for the services of relief organizations. For hunger relief, the food insecurity levels or poverty levels can serve as an estimate of the demand (Mohan et al., 2013). Disaster-induced demand is even more difficult to quantify due to the sudden nature of disaster strikes. Thus, relief items are pushed to some locations in anticipation of a disaster and pulled to other locations when the need arises (Whybark, 2007). Demand for supplies also vary greatly depending on the type and the impact of the disaster, demographics, and socio-economic conditions of the affected area (Balcik and Beamon, 2008). Balcik and Beamon (2008) categorized emergency relief items into two main groups: Type 1 and Type 2 items. Type 1 items include tents, blankets, tarpaulins, jerry cans, and mosquito nets; they are critical items for which the demand occurs once at the beginning of the planning horizon. Type 2 items are items that are consumed regularly and whose demand occurs periodically over the planning horizon. Examples of Type 2 items include food and hygiene kits. As a result of the limited supplies in humanitarian relief operations, unsatisfied demand is very common.

Another major challenge faced by relief operations is the allocations of resources equitably. Equitable distribution helps eliminate wastage of food due to excess food on one hand and unmet demand due to insufficient food on the other hand. Because demand is not fully satisfied, resources have to be distributed equitably so that everybody gets a fair share of the resources. Most supply allocation problems from literature are formulated as a multi-objective linear programming problems where the reduction or increment of desirable outputs and inputs such as cost minimization, minimization of travel time, and maximization of satisfied demand are the objectives (Tzeng et al., 2007; Davis et al., 2013). The constraints could be budget, capacity and time among many others.

2.3 Decision-making Process

Decision-making is the process of selecting a course of action among several alternative possibilities with the aim of a selecting the best action. The reason why making some decisions are harder than others is the level of uncertainty about the outcome of the decision in the present or the far future. The uncertainties arise as a result of (1) limitations in the ability to precisely model all the parameters and the variables related to the problem, (2) inability to accurately predict human behavior and (3) limited capacity to enumerate and process all the possible outcomes of the decision (Boularias, 2010).

Scientist and researchers have provided different types of decision support systems and models to help in the decision-making process to maximize some desired variables or arrive at an optimal solution. These decision-making tools can be qualitative or quantitative. Examples of the qualitative models are decision theories such as the rational and intuitive decision-making models. Quantitative models include mathematical models, decision trees, linear-programming models and Markov Decision Processes (MDPs) models.

Puterman (2009) describes a sequential decision-making process where the decision maker observes the environment or system and based on the state that the system is in, the decision maker chooses an action from a set of actions. The action produces an immediate reward whilst the system evolves to a new state at a different point in time according to some probability distribution. At this subsequent point in time, the system is in a different state with different set of actions to choose from and the whole process is repeated.

2.3.1 Quantitative decision-making models applied to inventory management

Quantitative decision-making models have been broadly applied to solve commercial inventory management problems and their applications ranges from determining reorder points for a single product to controlling complex supply chain networks (Puterman, 2009). Nahmias (2009) identified two fundamental decisions associated with inventory management: (1) when to order; which depends largely on the availability of the suppliers and the lead time variability and (2) how much to order; which largely depends on the demand and the desired service levels. The decision makers in commercial inventory management seek to maximize a profit index, which can be calculated as revenues minus ordering costs and inventory holding costs (Giannoccaro and Pontrandolfo, 2002). However the decision makers in relief inventory management seek to save human lives.

Decision-making in inventory management in the presence of random supply and demand can be very challenging with its obvious impacts of increasing operating costs and decreasing customer service levels. To cope with the random supply, commercial inventory managers have adopted sourcing from multiple suppliers (Mohebbi, 2003; Tomlin, 2006; Ahiska et al., 2013) or holding more inventories. The multiple suppliers and holding more inventory strategy cannot be directly applied to relief inventories due to the differences between these two types of inventories. Hence, the objectives of quantitative decision-making models for relief inventory control usually include identifying the quantity of supplies needed, when to intensify solicitation for supplies, finding optimal ways to increase their supplies and determining the equitable and effective distribution of the supplies.

2.3.2 Quantitative decision-making models applied to relief inventory management

The decision-making models for relief inventory control will be reviewed under the objective of the model. The objectives can be grouped under the following categories:

1. Determining the quantity of supplies needed, reorder points or safety stock levels
2. Equitable and effective distribution of supplies

2.3.2.1 Determining the quantity of supplies needed, reorder points/safety stock levels

Das and Hanaoka (2014) used a stochastic optimization model to support decision making in relief inventory management to identify the order quantity and reorder levels to prevent relief disruption following a large earthquake. The model incorporates the probability of a stock out per cycle, the expected shortage cost per cycle and the expected holding cost per cycle with the assumption of a stochastic demand and stochastic lead time, both uniformly distributed. The models considered two types of orders: a systematic order (normal delivery time) and an exigent order (delivered by expediting service). The exigent order was placed when the systematic order fails to arrive on time, though it incurs a higher cost to ensure the continuous flow of supplies.

Qin et al. (2012) proposed an inter-temporal integrated single-period inventory model to determine the optimal order quantity of emergency resources in an emergency situation such as flood incident. The emergency response was based on the perspective of integrating the "emergency management operational process" proposed by the Federal Emergency Management Agency (FEMA). They classified the emergency resources into response resources (resource 1) and recovery resources (resource 2), where demand for resource 2 is not only dependent on the shortage quantity of resource 1, but also on external stochastic factors. The model captured both the deterministic dependent relationship and stochastic dependent relationship between the shortage quantity of resource 1 and the demand quantity of resource 2 with the objective to minimize the expected loss related to all emergency resources. In order to reflect the dependent relationship in demand function of resource 2, they introduced a deterministic scalar multiplied with a stochastic variable. A genetic algorithm based simulation approach is used to solve the model.

Ozbay and Ozguven (2007) developed a time-dependent inventory control model for safety stock levels that could be used for the development of efficient pre-disaster and post-disaster plans. The proposed model attempted to determine the minimum safety stock so that the consumption of these stocked goods could occur without disruption for a given probability at minimum cost. Their research focused on obtaining such an effective humanitarian inventory management model using the "Hungarian Inventory Control Model"; a stochastic programming model, which was introduced by Prékopa. A solution procedure based on the concept of p-level efficient points (pLEPs) was also proposed.

Beamon and Kotleba (2006) also developed a stochastic inventory management model for a single item with irregular demand to determine the optimal order quantities and reorder points during a long-term emergency relief response. The model used the standard (Q, r) inventory policy that allows for two different order sizes, (Q_1, Q_2) and two different reorder levels, $(r_1,\ r_2)$. Q_1 was placed when the reorder level has reached r_1, a normal re-supply option. Q_2 was placed when the reorder level has reached r_2, expedited emergency re-supply. The expedited emergency supply incurs a higher fixed and per unit ordering cost than the normal orders. They assumed the demand to have a uniform distribution.

2.3.2.2 Equitable and effective distribution of supplies

Davis et al. (2013) determined the placement of supply within the supply chain network in preparation for natural disasters such as hurricanes, making use of short-term forecasts. They used a stochastic mixed integer linear programming approach, which considered both uncertainties in demand and supply. They developed a two-stage recourse model. The first decision stage was the preposition of supplies to minimize the total expected cost and the second stage was equitable distribution of supplies to minimize unmet demand.

Rottkemper et al. (2012) presented an inventory relocation and re-distribution model for decision-making to resolve the demand uncertainty problem in humanitarian relief for a single item. They considered a network model which comprised a global depot, a central depot and a number of regional depots. Transshipment between regional depots exists to allow effective relocation of inventory depending on demand surge during the relief action. A mixed-integer programming model was developed, which contained two objectives: minimization of unsatisfied demand and minimization of operational costs. To model uncertainty, demand was split into “certain” demand, which was known and “uncertain” demand, which occurred with a specific probability. Penalty costs were introduced for the unsatisfied certain and uncertain demand. A sensitivity analysis of the penalty costs was done to study the trade-off between demand satisfaction and logistical costs.

Bozorgi-Amiri et al. (2013) developed a multi-objective robust stochastic programming model for disaster relief logistics planning for earthquake scenarios under uncertainty. In their approach, demand, supply and the cost of procurement and transportation were considered as the uncertain parameters. Their model also considered uncertainty for the locations where the demands may arise. As well as the possibility that some of the pre-positioned supplies in the relief distribution center or the supplier might be partially destroyed by the disaster. The first objective was to maximize the ‘affected areas’ demand satisfaction levels and the minimize shortages in these affected areas. The second objective was to minimize the sum of the expected value and the variances of the total cost.

Lodree and Taskin (2008) proposed newsvendor variants that account for demand uncertainty as well as the uncertainty surrounding the occurrence of a disaster. The optimal inventory level was determined and compared to the classic newsvendor solution. The difference was interpreted as the insurance premium associated with proactive disaster-relief planning. The insurance premium was the additional costs incurred based on an order quantity that takes disruption into consideration. The insurance policy framework represented a practical approach for decision makers to quantify the risks/reward tradeoff associated with inventory decisions related to preparing for emergency relief efforts.

2.4 Markov Decision Process (MDP)

A Markov Decision Process (MDP) named after Andrey Markov (Markov, 1913) is a sequential decision making stochastic process that is used to study complex systems. They are generally characterized by five elements; decision epochs, states, actions, transition probabilities and rewards (Puterman, 2009). The decision maker's environment is often modeled as a dynamical system with different states. Decisions are made at the decision epochs (points in time), which can be discrete or continuous. Given such a system, the goal of the decision maker is to choose actions based on some decision rules that will move the system to a desirable state. A collection of the decision rules are called policies and the goal of the decision maker is to select the optimal policy that will maximize the total expected reward.

Specifically, the system has N number of states and at each decision epoch, $t \in T$, where T denotes the set of decision epochs, the process is in a current state $i \in S$ at time t; S is the state space which is a set of all possible states the system can occupy. As a result of the decision maker choosing any action a, from the action space, A_s, the process moves to the next state $j \in S$ or remains in the same state at time, $t + 1$. The decision maker receives a corresponding reward $r_{i\,j}$, which could be a gain or a loss, where R is the reward matrix with elements r_{ij}. The probability that the system moves from the current to the next state is influenced by the chosen action and it is given by the state transition probability, p_{ij}, where P is the transition matrix with elements p_{ij}. Transition to the next state j depends on the current state i and the decision maker's action a given that i and a are conditionally independent of all previous states and actions.

2.4.1Applications of MDPs to inventory management

The applications of MDPs in relief inventories are limited but have been greatly applied to commercial inventories. In commercial inventories, they are used to determine optimal order quantities and reorder points. The state of the system, which is a function of the inventory position, is viewed periodically or randomly according to an inventory review policy. Actions correspond to the amount of stock to be ordered with not ordering being a possible action (Giannoccaro and Pontrandolfo, 2002). The reward is to minimize the total expected costs, which include fixed ordering cost, unit ordering cost, holding cost, backordering penalty cost and lost sales cost.

Silbermayr and Minner (2014) studied a single item inventory system, with a buyer facing Poisson demand. The buyer can procure from a set of potential suppliers who are not perfectly reliable. Each supplier was considered to be fully available for a certain amount of time (ON periods) and then breaks down for a certain amount of time during which it can supply nothing at all (OFF periods). The problem was modeled as a Semi-Markov decision process (SMDP) where demands, lead times and ON and OFF periods of the suppliers are stochastic. The state of the system was defined as the inventory level, the number of outstanding orders with each supplier and the status of respective supplier. The actions corresponded to whether to place a new order, the quantity to order, and which suppliers to assign the order to. The objective was to minimize the buyer's long run average cost, including purchasing, holding and penalty costs.

Ashika et al. (2013) used a discrete-time Markov decision process (DTMDP) to model the supply interruption problem in order to find the optimal ordering policies that would minimize the total expected cost. They considered an infinite-horizon, single-product, and periodic review inventory system for a retailer who had adopted a dual sourcing strategy in the presence of stochastic demand. Amongst the two suppliers, one was perfectly reliable while the other was not but offered a lower price. The system states were defined as the inventory level and the unreliable supplier status, which could be either *up or down.* The actions are the quantities to order from each supplier.

Warsing Jr et al. (2013) formulated a discrete-time Markov process (DTMP) model with a finite state space for a single-item, single-site inventory system, operating under a periodic-review, with stochastic demand and imperfect supply. The supplier was not entirely reliable. Each order was represented as a Bernoulli trial. With probability α the supplier delivered the current order and any accumulated backorders at the end of the current period. With probability $1-\alpha$, the supplier would fail to deliver. Their objective was to determine the optimal base-stock level and minimize the long-run average system cost per period. The states of the system were the inventory level and actions are the order quantities.

Wang et al. (2010) modeled a multi-period newsvendor problem with partially observed supply-capacity information, which evolved as a Markovian Process (POMDP). Their objective was to determine an optimal purchasing policy that minimized the total cost using a dynamic programming formulation. In their model the supply capacity is fully observed by the buyer when the capacity is smaller than the buyer's ordering quantity. But partially observed when the capacity is greater than the buyer's ordering quantity. The available capacities of the supplier were the states of the system and actions were the buyer's order quantities.

Tomlin (2006) also investigated an infinite horizon, periodic inventory model with two suppliers; reliable and unreliable with complete backlogging of unmet demand using a discrete-time Markov process (DTMP). The objective was to determine the optimal sourcing strategy and the optimal order quantity whilst minimizing the long-run average cost. The states of the system were the unreliable supplier status, which could be *up* or *down* and were represented as a Markov process. The actions were the quantities to order from each supplier. They investigated two scenarios of demand: stochastic but stationary demand and deterministic demand.

Giannoccaro and Pontrandolfo (2002) look at the coordination of inventory policies adopted by different supply chain actors, such as suppliers, manufacturers and distributors. They modeled the problem as a Semi Markov Decision Process (SMDP) and a reinforcement learning (RL) algorithm is used to determine a near optimal inventory policy under an average reward criterion. The objective was to ensure the smooth flow of materials, meet customer demand responsively whilst minimizing the total supply chain costs. The states of the system were the inventory position at each stage of the supply chain. Actions at each stage range from ordering nothing up to ordering the maximum quantity. The quantity to order equaled to the stock point capacity plus the current backorder plus the estimated consumption during the transportation lead time minus the stock on hand.

2.5 Summary of Literature

Much of the literature surrounding humanitarian relief management has been centered on disaster related issues as these are considered to be very serious and devastating situations. But the issue of hunger, which is a long-term humanitarian issue, has received very little research despite the compelling evidence of increasing food insecure households. Hence, there is a need for more research in this area to help hunger-relief organizations to improve their supply chains, especially procurement of supplies and managing their inventories to increase their service levels. Nevertheless, the irregular supply patterns and other constraints inherent in relief inventories present unique challenges to relief inventory managers.

Commercial inventory managers effectively improve their operations using quantitative decision-making modeling for inventory management systems. The techniques used include linear programming, stochastic programming, mixed integer linear programming, genetic algorithms and Markov decision making processes. These techniques have also been used to model relief inventories. However, quantitative decision-making models for relief inventory management have been centered on linear and stochastic programming techniques that considered demand uncertainty. Just a few of these techniques considered uncertainties in the supply as well. Table 2 shows the summary of some quantitative decision-making models for relief inventory control that have been used to solve a variety of real-life problems.

Though MDPs are very powerful analytical tools that have been used in many instances to solve complex problems with uncertainties, they have not been used to model relief inventories. Most of their applications in inventory management have been widely centered on commercial inventories to deal with the problem of unreliable suppliers to determine optimal sourcing strategies and optimal order quantities that minimize overall cost. Table 3 shows the summary of some application of MDPs to commercial inventory management problems.

Table 2

Quantitative decision making models for relief inventory control

Authors	**Type of relief**		**Objectives**				**Uncertainty**		**Model**
	Disaster relief	**Hunger relief**	**Cost minimization**	**Quantity of supplies needed**	**Equitable distribution of supplies**	**Soliciting for supplies**	**Demand**	**Supply**	
(Beamon and Kotleba, 2006)	●			●			●		Stochastic
(Bozorgi-Amiri et al., 2013)	●		●		●		●	●	Stochastic

Table 2

Cont.

(Das and Hanaoka, 2014)	●			●			●		Stochastic optimization
(Davis et al., 2013)	●		●		●		●	●	Stochastic mixed integer linear programming
(Ozbay and Ozguven, 2007)	●		●	●			●		Stochastic
(Jr and Taskin, 2008)	●			●	●		●		Mathematical
(Qin et al., 2012)	●			●			●		Mathematical model & Genetic algorithm
(Rottkemper et al., 2012)	●		●		●		●		Mixed-integer programming

Table 3

Applications of MDPs to inventory commercial management

Author(s)	**Model**	**Problem**	**Decisions**	**State**	**Objective function**
(Ahiska et al., 2013)	DTMDP	Unreliable supply & stochastic demand	Ordering quantities	Inventory level, unreliable supplier status	Minimizes the total expected cost
(Warsing Jr et al., 2013)	DTMDP	Imperfect supply & stochastic demand	Base-stock level	Inventory position	Minimize the average system cost per period
(Wang et al., 2010)	DTMDP	Unreliable supply	Purchasing policy	Supplier's capacity	Minimizes the total cost
(Tomlin, 2006)	DTMDP	Unreliable supply	Sourcing strategy	Unreliable supplier status	Minimize the long-run average cost
(Giannoccaro and Pontrandolfo, 2002)	SMDP	Unreliable supply	Ordering policy	Inventory position	Minimize total supply chain costs
(Silbermayr and Minner, 2014)	SMDP	Unreliable supply	Ordering policy	Inventory position	Minimizes the total cost

2.5.1 Research contribution

This research contributes to the literature by simultaneously considering the objectives of increasing service levels and equitable distribution of supplies to obtain optimal policies for donation solicitation and distribution for a hunger-relief organization. The study also highlights the pound per person in poverty (PPIP) as measure of distribution of supplies. The approach presented in this research uses the Discrete Time, Discrete State (DTDS) Markov decision-making (MDP) model. The DTDS MDP model will find an optimal allocation policy to equitably distribute food items to aid recipients. In addition estimate how long it takes to meet the PPIP criterion of 75 pounds per person set by Feeding America. The model can be used for benchmarking the performance of hunger-relief organizations in their efforts to meet the needs of the people they serve.

CHAPTER 3

Methodology

3.1 Problem Overview

More than 70% of the supplies to the Food Bank of Central & Eastern North Carolina (FBCENC) are donations from individuals and organizations. These supplies are subject to significant variations. Furthermore, the demands that the FBCENC needs to satisfy normally exceeds these supplies that come in. The FBCENC has six branches namely, Wilmington (W), Durham (D), Raleigh (R), Sandhills (S), Greenville (G), and New Bern (NB). These branches serve as warehouses for the FBCENC. The warehouses sort the supplies, conduct quality assessment and then store the supplies for distribution. The food bank branches transfer supplies among themselves when it becomes necessary. The Raleigh branch serves as the main warehouse; it receives a lot of donations. Thus, it transfers most of the supplies it receives to the other branches. Figure 4 illustrates how supplies flow in the FBCENC network.

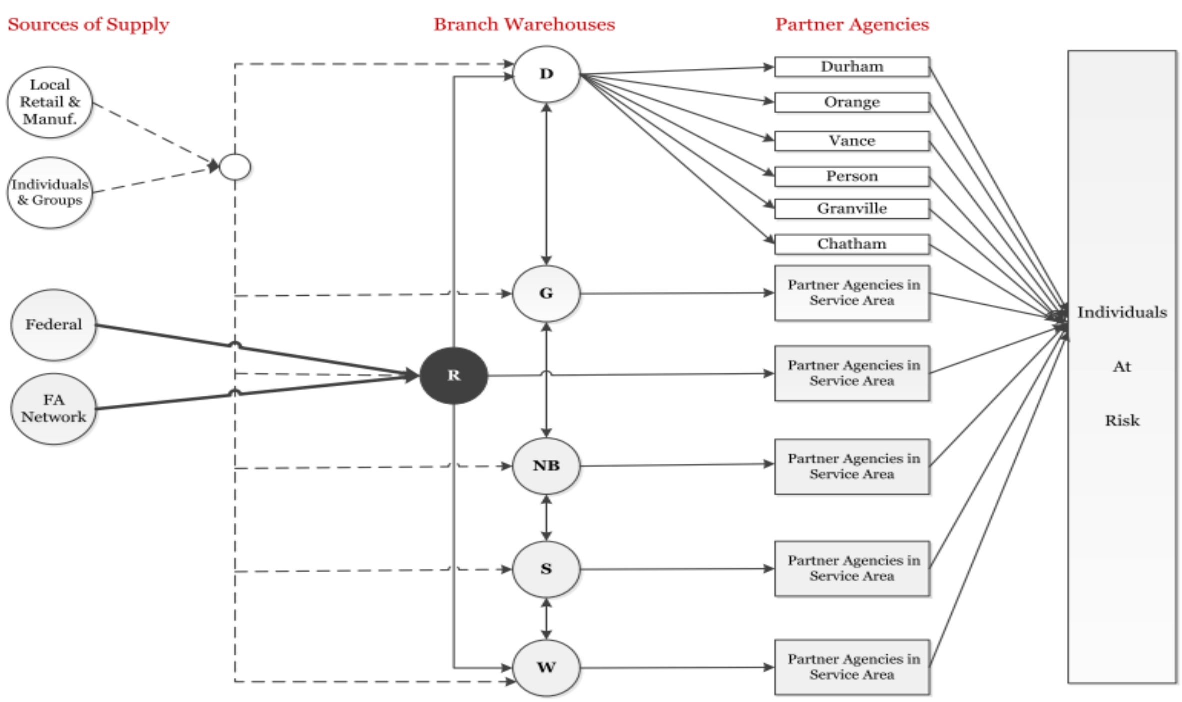

Figure 4. Supply flow in the FBCENC network.

Each branch (warehouse) has specific counties that it distributes supplies to. These counties further distribute the supplies to individuals at risk of hunger in these counties. Feeding America expects FBCENC to distribute food across counties such that the pounds of food distributed per person in poverty (PPIP) is at least 75 over a 12 month period. Counties whose PPIP is below the 75 pounds target are considered to be underserved. The management at FBCENC wants to use the performance indicator (PPIP) proposed by Feeding America to measure how the FBCENC performs over time.

This chapter focuses on the formulation of a discrete time, discrete space (DTDS) Markov Decision Process (MDP) to achieve the following objectives:

1. Find an optimal supply allocation policy that maximizes equity in the distribution of supplies using the PPIP criterion of 75;
2. Estimate the number of underserved counties and the unsatisfied demands.

3.2 Model Assumptions

The inventory system considered is a single item inventory system with periodic review. Before we proceed to provide a mathematical formulation of the problem, the following general assumptions are made about the proposed DTDS Markov Decision model:

1. The state of the system is available inventory of the warehouse at the beginning of each month;
2. Donations follow a normal distribution and occur along the time period;
3. The poverty population in a given county serves as an estimate of the demand for that county;
4. Demands are deterministic, occur along the time period and are fulfilled before the beginning of the next time period;

5. Demands are met using the available inventory and the donations according to a predefined allocation rule;
6. Distribution cannot exceed demand for a given county;
7. Transfers from other branches (transfer-in) follow a normal distribution and occur at the end of the time period;
8. There is no reallocation of supplies after distribution;
9. Transfer-in is added to what is left in inventory at the end of each period and carried over to the next period.

3.3 Model Formulation

3.3.1 Notations and definitions

Unless otherwise stated, the following parameter definitions are used throughout the remainder of the model formulation.

Table 4

Model parameters and their definitions

Notations		Definitions
Sets	V	Set of all possible system states
	C	Set of counties to be served
	A	Set of allocation rules $A = \{a_1, a_2 \ldots a_N\}$
	T	Time periods with $t \in \{1 \ldots \tau\}\tau < \infty$
State variables	v_t	Available inventory at time t , $v_t \in V$ (measured in pounds)
Random variables	X_t	Food donations at time t with realization $x_t \in X_t$
	Y_t	Transfers of food from other locations at time t with realization $y_t \in Y_t$

Table 4

Cont.

Decision variables	k_{ct}^{a}	Pounds of food distributed to county c at time t given allocation a
	f_c	Fraction of available inventory allocated to county $c \in C$
Reward variables	$r_c(a)$	Pounds of food distributed per person in poverty in county $c \in C$ under allocation rule $a \in A$
Other variables	$\tilde{v}_t$	Percentage deviation from mean available inventory
	$\tilde{x}_t$	Percentage deviation from mean donation amount
	$\tilde{y}_t$	Percentage deviation from mean branch transfer
Parameters	P_c	Poverty population in county $c \in C$
	d_c	Demand for county $c \in C$ at time t
	H_c	History of total distribution over the previous 11 months to county c
	μ_I	Average inventory in pounds
	μ_D	Average donation in pounds
	μ_B	Average branch transfer in pounds

3.3.2 Sequence of events in the model

At the beginning of the month, the state of the system (available inventory) is known, donations occur along the period and demands also occur along the period. Received donations are added to the current available inventory to satisfy the demands. The total inventory is then distributed to the counties based on the corresponding allocation decisions. The left over inventory is noted after the demands have been served. Transfer-in from other branches come in at the end the period and it's added to the left over inventory. The system transitions to the next state, which is the remaining inventory plus the transfer-in. Figure 5 illustrates how the system moves from one state to the next.

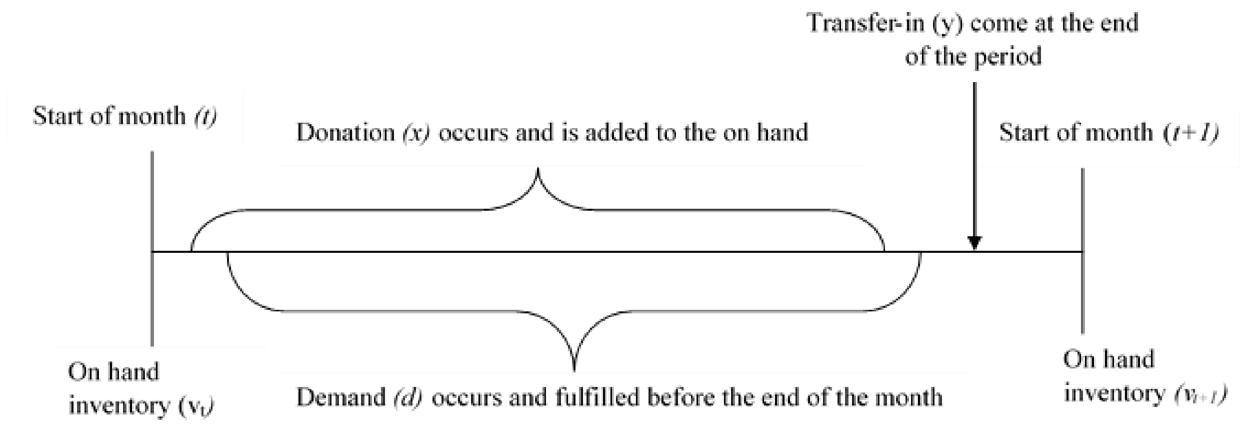

Figure 5. Timing of events in the model.

3.3.3 Decision epoch

Allocation decisions are made on a monthly basis before the beginning of the next month. Within a period of one month the incoming donations are assumed to be available for distribution instantaneously. This is because the warehouse would have been able to sort the supplies and review it for quality in accordance with the standards for food safety and distribution practices of the food bank within this time period. The model is a finite time horizon model and the set of finite time periods is denoted as $T = \{1,2, \ldots, \tau\} \quad \forall\, \tau < \infty$.

3.3.4 State of the system

The state of the system is the available inventory of the warehouse at the beginning of each time period. Available inventory represents the supplies (measured in pounds) in the warehouse. Based on the data received from FBCENC the available inventory values are very large and this can be any value within the ranges of 100, 000 to 800,000 pounds. Since the model is a discrete state MDP, the state space is discretized using the discretization procedure described in section 3.3.4.1 below. It should be noted that the system state space denoted by $V = \{1,2, \dots, M\}$ where $1,2, \dots M$ are pseudo states that represent the discretized form of the actual available inventory values.

3.3.4.1 Discretization procedure

Discretization entails the process of transferring continuous data or models into discrete equivalents. While the number of continuous values for a data can be infinitely many, the number of discrete values is often few or finite. This makes discrete values easier to understand, use, and explain.

3.3.4.1.1 Mean percentage deviation

To reduce and simplify the continuous data, a heuristic approach called the mean percentage deviation shown in equation (1) below is used to calculate the percentage deviation of the actual value α from the mean value, μ.

$$Mean\ Percentage\ Deviation = \frac{\alpha - \mu}{\mu} \times 100\% \qquad (1)$$

Thus, any continuous data can be represented with the mean and the mean percentage deviations. The mean percentage deviations can be grouped into intervals and one can reconstruct actual values from the mean and mean percentage deviation intervals with some errors due to the grouping.

3.3.4.1.2 Binning

Binning is one of the simplest methods used to discretize continuous data. Based on the range of the original continuous data, sub-ranges called bins are created. The binning technique groups values into bins. The bins can be created by equal-width. Let α_{min} and α_{max} be the minimum and the maximum percentage deviation of the actual values. Thus the set of percentage deviation of the actual values α is bounded by the range $\alpha_{min} \leq \alpha \leq \alpha_{max}$ where $\alpha_{min} > -\infty$ and $\alpha_{max} < \infty$. The percentage deviation of the actual values α are grouped into bins taken into consideration α_{min} and α_{max}. Let M be the number of bins, which are numbered, 1 through M, $m \in M$. Also let Δ_α be the bin width given by $\Delta_\alpha = \frac{(\alpha_{\max} - \alpha_{min})}{M}$. Then, the range, R_m, of the mth bin is as shown in equation (2).

$$R_m = (\ \alpha_{min} + (m-1)\Delta_\alpha \qquad \alpha_{min} + m\Delta_\alpha\ ] \tag{2}$$

It should be noted that, the choice of the number of bins M is discretional. The lower and the upper ranges for R_m are given by $R_1 = (-\infty \quad \alpha_{min} + \Delta_\alpha\ ]$ and $R_M = [\alpha_{min} + (M-1)\Delta_\alpha \qquad \infty\ )$ respectively. This boundary ranges are essential to cater for unknown data points that might fall outside the predefined domain, $[\alpha_{min}\ \ \alpha_{max}]$ during the lifetime of the model. In our approach, the values to bin are the percentage deviations from the mean. Consequently, each percentage deviation value belongs to one of the bins.

3.3.4.1.3 Mapping

A one to one mapping relation is used to associate the bins with distinct discrete values. Thus a specific bin is replaced by a discrete value such as 1, 2, or 3.

3.3.5 State transitions and transition probability

The events that cause a transition from one state to the next are:

1. Donation, $x_t \in X_t$, which is stochastic with CDF $\Phi_x(\bullet)$ and follows a normal distribution
2. Transfer-in, $y_t \in Y_t$, which is stochastic CDF $\Phi_y(\bullet)$
3. Distribution to aid recipients, k_{ct}^{a} given allocation decision $a \in A$
4. The available inventory at time t, v_t

Given the above transition parameters, the available inventory in the next time period v_{t+1} can be computed using the transition function shown in equation (3).

$$v_{t+1} = \left[v_t + x_t - \sum_{c \in C} k_{ct}^a \right]^+ + y_t \tag{3}$$

In equation (3) $v_t = (1 + \tilde{v}_t)\mu_I$, $x_t = (1 + \tilde{x}_t)\mu_D$, $y_t = (1 + \tilde{y}_t)\mu_B$ where v_t, x_t and y_t are the actual values of the available inventory, donation and transfer-in (measured in pounds) respectively. To obtain a distinct discrete value that represents v_{t+1}, the mean percentage deviation $\tilde{v}_{t+1}$ is calculated and the result binned. The discrete value associated with the bin is the pseudo state that represents the available inventory in the next time period, v_{t+1}.

The probability that the system moves from the current state v_t, to the next state v_{t+1}, is influenced by the donation and transfer probabilities, which are assumed to be normally distributed. The transition probabilities are shown in equation (4).

$$p(v_{t+1}|v_t, a) = \begin{cases} p(y_t) \sum_{x_t \le K_{t-v_t}} p(x_t) & v_{t+1} = y_t \\ p(x_t)p(y_t) & v_{t+1} = y_t + v_t + x_t - K_t; K_t - v_t < x_t \\ 0 & otherwise \end{cases} \tag{4}$$

It should be noted that $0 \le p(v_{t+1}|v_t, a) \le 1$, $\sum_{v_{t+1}} p(v_{t+1}|v_t, a) = 1$ and $K_t = \sum_{c \in C} k_{ct}^a$.

Based on the discretization procedure and given $x_t \in R_m$ or $y_t \in R_m$, then $p(x_t)\ or\ p(y_t) = \phi(R_m^+) - \phi(R_m^-)$, where R_m^+ is upper bound of the bin range and R_m^- is the lower bound of the bin range.

3.3.6 Allocation rules

Allocation rules correspond to the actions that the decision maker can choose based on the state of the system. Allocation rules have been widely used in commercial inventory management. An example is the case of ‘one warehouse and multiple retailers’ scenario (OWMR). The warehouse orders from an external supplier with unlimited capacity and the retailers order from the warehouse. The warehouse can fill up orders only when stock is available. Some of the allocation rules identified under the OWMR system are:

a. **Myopic/ Last minute stock allocation;** the warehouse postpones the decision of how much to allocate to each retailer until the moment (last minute) of shipment or (last minute) of delivery. The decision of how much to distribute is based on the updated retailer's inventory information available at those times (Howard and Marklund, 2011).

b. Allocate based on the sequence in which retailer orders arrive to the warehouse;

   a. First In First Out (FIFO) /First Come First Served (FCFS);

   b. Last In First Out (LIFO)/ Last Come First Served (LCFS);

c. **Fixed allocation;** each retailer receives a predetermined fraction of goods in each period (Kempf et al., 2011).

d. **Proportional allocation;** each retailer receives proportion of goods based on their share of the total demand (Kempf et al., 2011).

For this model the decision maker uses the Allocation rules to distribute supplies to the counties. These decisions are formulated in three unique ways as follows.

3.3.6.1 Serve largest demand first (SLDF) – Rule 1

With the SLDF, the decision maker serves the county with the largest demand and proceeds down the hierarchy to serve the next larger demand and eventually serves the least demand last based on what is left after previous distributions. Thus, using rule 1, the distribution to county c with deterministic demand, d_c at time t is given by equation (5).

$$\text{SLDF } (rule\ a_1)\text{: } k_{ct}^{a_1} = min\left(max\left(v_t + x_t - \sum_{i \in F_t^*} k_{it}\, , 0 \right), d_c \right) \tag{5}$$

In equation (5), $F_t^* = \{c' \in C | d_{c'} > d_c\}$. In other words, F_t^* is a set of all previous deterministic demands $d_{c'}$ that have been served. It should be noted that at the beginning of the process, there are no previously served demands.

3.3.6.2 Serve smallest demand first (SSDF) – Rule 2

For SSDF, the decision maker serves the county with the smallest demand and proceeds up the hierarchy to serve the next smaller demand and eventually serves the highest demand last based on what is left after previous distributions. Thus, using rule 2, the distribution to county c with the deterministic demand, d_c at time t is given by equation (6).

$$\text{SSDF } (rule\ a_2):\ k_{ct}^{a_2} = min\left(max\left(v_t + x_t - \sum_{i \in F_t^*} k_{it}, 0 \right), d_c \right) \tag{6}$$

In equation (6), $F_t^* = \{c' \in C | d_{c'} < d_c\}$. In other words, F_t^* is a set of all previous deterministic demands $d_{c'}$ that have been served. It should be noted that at the beginning of the process, there are no previously served demands.

3.3.6.3 Proportional allocation – Rule 3

Rule 3 uses the proportional allocation approach by (Kempf et al., 2011) to distribute supplies to the counties such that each county receives supplies based on the ratio of their poverty population to the total poverty population. Using rule 3, the distribution to county c with deterministic demand, d_c at time t is given by equation (7).

$$\text{Proportional allocation } (rule\ a_3):\ k_{ct}^{a_3} = min\left(\frac{P_c}{\sum_{c \in C} P_c} * (v_t + x_t), \qquad d_c \right) \tag{7}$$

3.3.6.4 Fixed allocation – Rule 4

For fixed allocation, the decision maker distributes fixed amount of supplies to each county irrespective of their demand sizes. Using rule 4, the distribution to county c with deterministic demand, d_c at time t is given by equation (8).

$$\text{Fixed allocation (rule } a_4\text{);}\quad k_{ct}^{a_4} = min(f_c(v_t + x_t),\quad d_c) \tag{8}$$

The variable, f_c in equation (8) is the fraction of the total inventory allocated to county, c such that $0 < \sum_{c=1}^{N} f_c \leq 1$. These fractions may be obtained from observational records or may be at discretion of the decision maker and may or may not reflect the poverty population of the counties.

3.3.7 Reward determination

The FBCENC desires to equitably distribute the supplies to the aid recipients through the warehouses (branches) and potentially achieve a long-term goal of meeting the PPIP target of 75 pounds over a 12-month period. In other words, equity in distribution is to ensure that each person in poverty in the various counties receives equal share of the pounds of food distributed. The reward is therefore an objective function that maximizes equity in distribution.

There are several techniques that are used to measure equity. These include but are not limited to difference between the maximum and minimum values, variance, coefficient of variation, sum of absolute deviations, maximum deviation, and mean absolute deviation (Marsh and Schilling, 1994). Equity is maximized by minimizing these measurements. This research rather aims at formulating a measurement (reward) that when maximized, maximizes the equity as demonstrated in the subsequent sections.

3.3.7.1 Pounds distributed per person in poverty (PPIP)

The PPIP is the ratio of what has been distributed over a 12-month period to the poverty population in that county. Thus, the PPIP associated with each county c for given action $a \in A$, can be computed by equation (9).

$$r_c(a) = \frac{k_{ct}^{a} + H_c}{P_c} \tag{9}$$

Let $PPIP_t$ be the target PPIP as set in accordance with the Feeding America performance indicator as the benchmark to measure the performance of the food bank branches. Thus, if the $r_c < PPIP_t$ the county is considered to be under-served. On the contrary if $r_c > PPIP_t$, the county is said to be over-served. Otherwise, the county is well-served.

3.3.7.2 Measure of equity

The mean absolute deviation (Δ_c) of the pounds per person in poverty for each county's is used as the central piece to measure equity. Generally, the mean absolute deviation is calculated as shown in equations (10) and (11). Let,

$$\bar{r}(a) = \frac{1}{|C|}\sum\nolimits_{c \in C} r_c(a) \tag{10}$$

$$\Delta_c(a) = \sum_{c \in C} \frac{|r_c(a) - \bar{r}(a)|}{\bar{r}(a)} \tag{11}$$

Consequently, the greater the value of the mean absolute deviation, the less the equity. On the other hand, perfect equity is achieved when $\Delta_c = 0$. To ensure that large mean absolute deviation corresponds to large equity, equations (11) is rewritten to obtain equations (12).

$$\Delta_c(a) = 1 - \sum_{c \in C} \frac{|r_c(a) - \bar{r}(a)|}{\bar{r}(a)} \tag{12}$$

Thus, maximizing equation (12), maximizes the equity. In this case, perfect equity is achieved when $\Delta_c(a) = 1$. This formulation has the advantage of being able to track whether perfect equity is obtained over a given time horizon.

3.3.7.3 Expected immediate reward

The immediate reward, which is the expected reward for state v_t under allocation rule a is shown in equation (13)

$$q(v_t, a) = E_{X_t, Y_t}\left[1 - \sum_{c \in C} \frac{|r_c(a) - \bar{r}(a)|}{\bar{r}(a)}\right] \tag{13}$$

3.3.8 Optimal policy determination

A policy provides the decision maker with the decisions to make at all decision epochs as a function of the state. Bellman (1954) applied the “Principle of Optimality” to Markov Decision Processes; to determine the optimal policy that maximizes or minimizes a reward criterion. This principle states that; given a current state, an optimal policy for the remaining states is independent of the policy adopted in the previous states. The optimal policy and the corresponding reward can be determined by three methods: Policy-iteration (Howard, 1960), value-iteration (Howard, 1960) and linear programming. For finite horizon problems the value-iteration which is the same as the Bellman equation is used.

3.3.8.1 Bellman’s equation

Let $U_t(v_t, a)$ represent the total expected reward at time t, starting from state v_t, if allocation rule a is used. The optimality equation can be formulated as shown in Equation (14).

$$U_t(v_t, a) = max_{a \in A}\left[q(v_t, a) + \sum\nolimits_{v_{t+1} \in V} p(v_{t+1}|v_t) U^*_{t+1}\,(v_{t+1})\right] \tag{14}$$

3.3.8.2 Backward induction algorithm

The Backward induction is the process of reasoning backwards in time, thus considering the last time a decision might be made and choosing what to do in that time. Using this information, the decision maker can then determine what to do at the subsequent times. This process continues backwards until the decision maker has determined the best action for every possible state at every point in time. The backward induction algorithm(Puterman, 2009) is used to determine the optimal policy as shown in the steps below.

1. Set $t = \tau$ and $U^{(k)}(v_{t+1}, \tau) = 0$ Substitute $t - 1$ for t and compute equation (15) show below. Equation (16) gives the argument that maximizes $U_t(v_t, a)$.

$$U_t(v_t, a) = max_{a \in A}\left[q(v_t, a) + \sum\nolimits_{v_{t+1} \in V} p(v_{t+1}|v_t)U^*_{t+1}\,(v_{t+1})\right] \tag{15}$$

$$Set\ a^*(v_t) = arg\, max_{a \in A}\left[q(v_t, a) + \sum\nolimits_{v_{t+1} \in V} p(v_{t+1}|v_t)U^*_{t+1}\,(v_{t+1})\right] \tag{16}$$

2. If $t = 1$, stop. Otherwise return to step 2.

The use of the Optimality equation will determine the optimal policy, for each state, at each time period and the optimal reward associated with this policy.

3.3.9 Estimation of underserved counties

Unsatisfied demand is the amount of additional supplies that is needed by the counties to meet the target PPIP, $PPIP_t$. The unsatisfied demand for each county c can simply be estimated as $(PPIP_t - r_c) \times P_c$.

CHAPTER 4

Data Analysis and Experimental Design

4.1 Introduction

This chapter focuses on preprocessing of the Food Bank of Central and Eastern North Carolina (FBCENC) data. This is necessary in order to transform the FBCENC data into a format that can be used with the Discrete Time, Discrete State (DTDS) Markov Decision Process (MDP) model, which was formulated in Chapter 3. This chapter then proceeds to outline an experimental setup that attempts to run different real-life or near real-life scenarios to demonstrate how the model adapts to variations in the demands in relation to the limited supply and examine their impacts on the objectives of the model. These objectives are re-stated as follows:

1. Find optimal supply allocation policy that maximizes equity in the distribution of supplies to counties by using the PPIP criterion;
2. Estimate the number of underserved counties and the unsatisfied demands.

4.2 Data

4.2.1 FBCENC data

The FBCENC data contains records of donation, distribution, and transfer transactions from 2006 to 2014. These records are grouped into fiscal years (FY) and stored in Microsoft Access Databases. A fiscal year begins from July to June of two consecutive years. For example, fiscal year 2006/2007 (denoted as FY0607) begins from July 2006 and ends on June 2007. The database has records of eight fiscal years (from FY0607 to FY1314).

4.2.2 FBCENC dataviewer software

FBCENC daily transaction records grow rapidly and are susceptible to data entry errors. As a result, the FBCENC routinely validates the data and makes adjustments whenever necessary to entries of perishable items and transfers (in or out). Developing predictive models for daily transactions of this nature is difficult if not impossible due to the large amount of noise in the data. It is also a challenge to quickly retrieve records for monthly data analyses. To reduce these bottlenecks, two design approaches are employed. First, all the FBCENC datasets are converted into Microsoft SQL database tables and are hosted on Microsoft SQL Server. Second, an interactive software called "FBCENC DataViewer" was developed as part of this thesis to query the database on the Microsoft SQL Server. This makes information retrieval from the database efficient and practical. Even though, the objectives of this thesis do not include the development of information retrieval system, it is worth providing a high-level description of how the developed software functions and how it contributes to the success of this thesis.

The FBCENC DataViewer provides an interface to interact, visualize and analyze the FBCENC data in an organized and meaningful format. The software has two major user interfaces (UIs). The first UI displays a set of mandatory fields with options from which the user must choose and submit a query to the database. There are five categories of records that the user can select from. These include, Distribution Records, Donation Records, Transfer Records, Waste Records and Custom Queries. The key fields that the user uses to query the databases are Fiscal Year, Fiscal Month, Branch Code, Storage Type and Product Type. A detailed description of what each of these categories and key fields mean is shown in the Appendix B. The second UI is a result page, which displays a table of results based on the query submitted from the first UI. The data can also be displayed in a graphical format using visualization options such as time series, bar chart or pie chart. In addition, the data displayed can easily be exported into Microsoft Excel for further data analyses. Figures 6 and 7 show examples of the first UI and the second UI respectively.

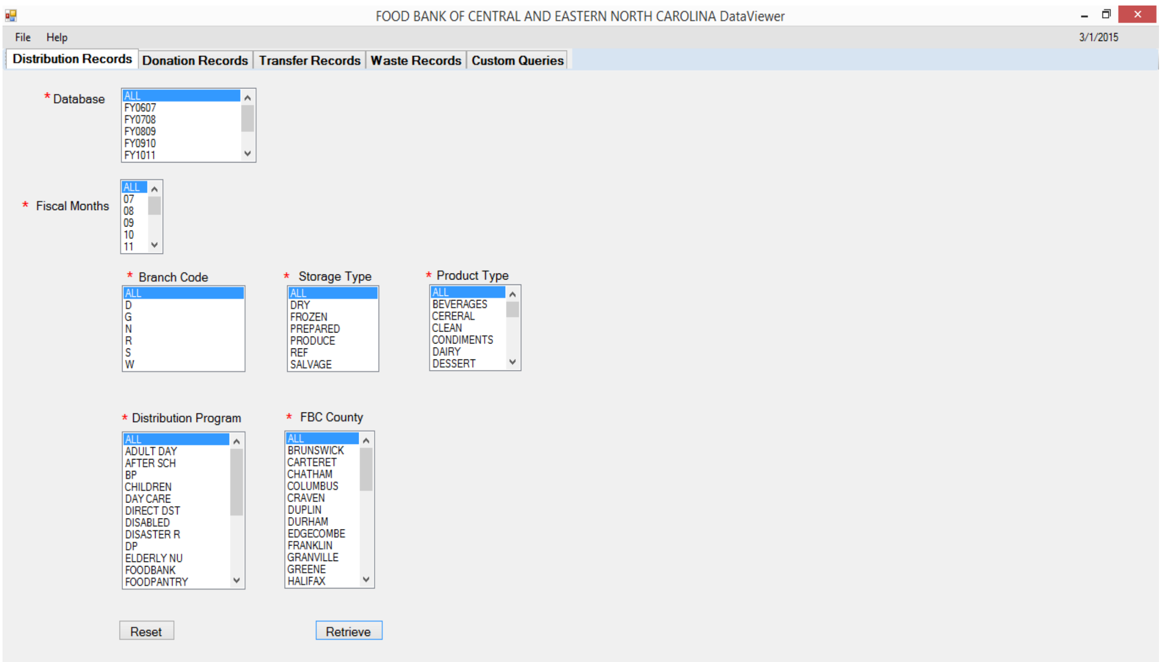

Figure 6. Interface showing how to retrieve data using the developed FBCENC DataViewer.

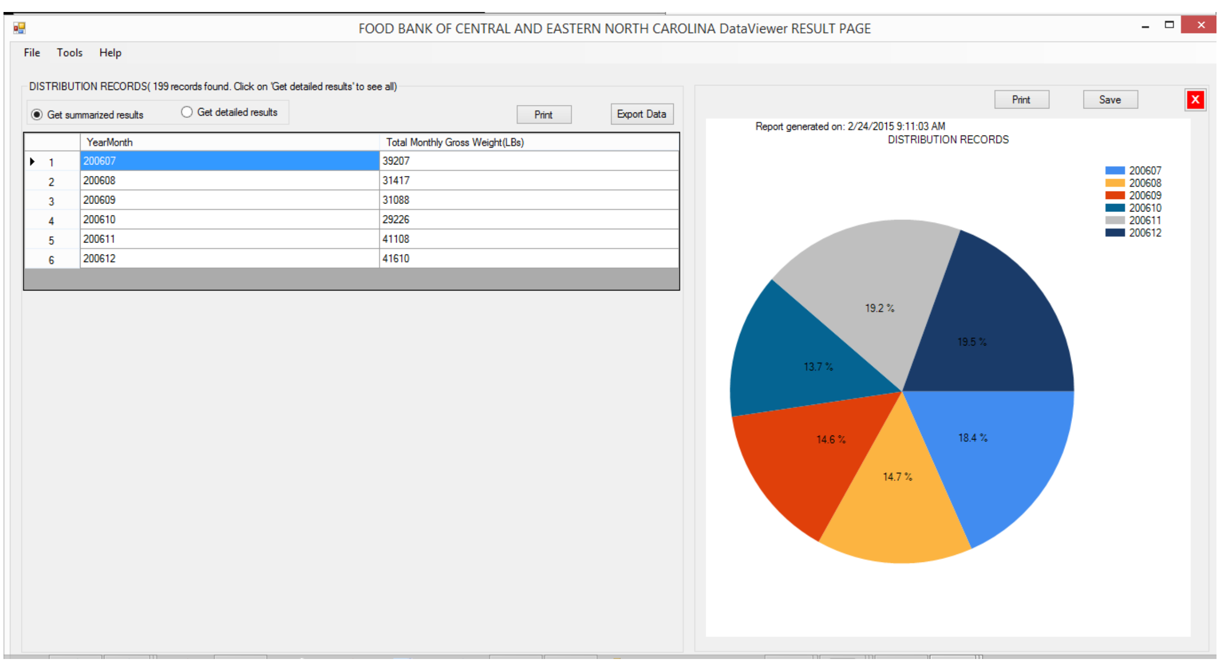

Figure 7. A user interface showing a sample result using the developed FBCENC DataViewer.

In what follows, we discuss how the FBCENC DataViewer is used to acquire the data needed to test our developed Discrete Time, Discrete State (DTDS) Markov Decision Process.

4.3 Data Retrieval using FBCENC DataViewer

The model formulated in the methodology considers a warehouse with a single item inventory system with monthly review. The Durham branch (warehouse) of the food bank is investigated with a focus on dry goods (all items classified under dry storage type). The developed FBCENC DataViewer described above was used to retrieve the relevant data from all the eight fiscal years for the Durham branch. The relevant data for this research are the donation, transfer and distribution records. The data is aggregated on a monthly basis showing the pounds of items received or distributed. Table 5 summaries the categories and key fields that were selected from the FBCENC database using the FBCENC DataViewer.

Table 5

Summarization of data

Data Fields	Description
FBCENC Records	Donations, Transfers and Distribution
Database	All fiscal years
Fiscal Month	All fiscal months
Branch	Durham
Storage Type	Dry storage

4.3.1 Durham branch

The Durham branch serves six counties namely Chatham, Durham, Granville, Orange, Person and Vance Counties. The poverty populations of these counties is obtained from the FBCENC's fair share program (Food Bank of Central and Eastern North Carolina, 2012). The fair share program is designed to ensure that each county receives food in proportion to its percentage of the overall need. Figure 8 shows the number of people living in poverty in each county. From Figure 8, Durham County has the largest poverty population, which is approximately 44% of the entire poverty population being served by the Durham branch.

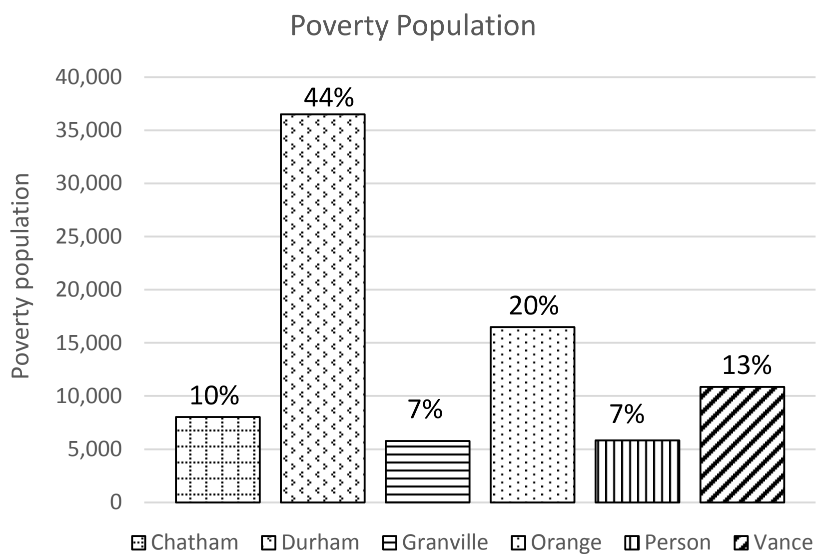

Figure 8. Counties poverty population.

Different counties have different poverty population in the Durham branch.

4.3.1.1 Donations

The Durham branch receives donations from various sources such as the Federal government through the Emergency Food Assistance Program (TEFAP), individual donors, groups, farmers, local manufacturers and retailers. Each fiscal year, the Durham branch receives approximately an average of 2,766,000 lb. of supplies as donations of which 56% are dry goods. Figure 9 illustrates the average amount of dry goods donated each fiscal year over the eight years.

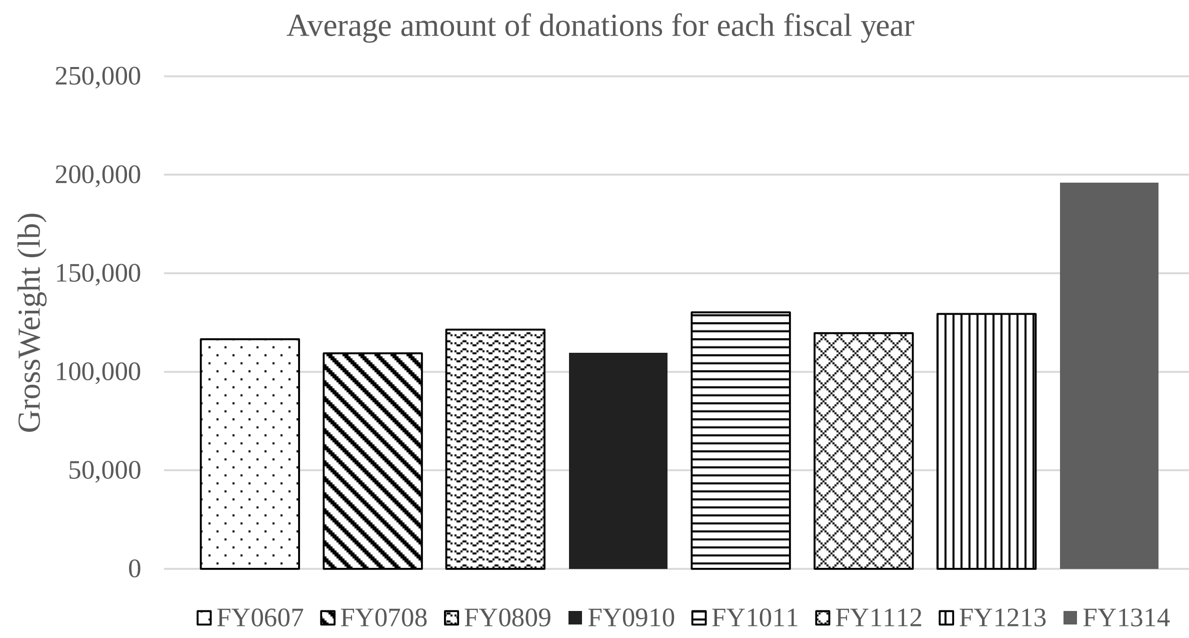

Figure 9. Average amount of donation of dry goods for each fiscal year.

Figure 10 shows a time-series graph of the monthly donations of dry goods over the eight fiscal years with average monthly donation of approximately 129,000 lbs. The minimum and the maximum donations received over the eight fiscal years are 30,000 lb. and 334,000 lb. respectively.

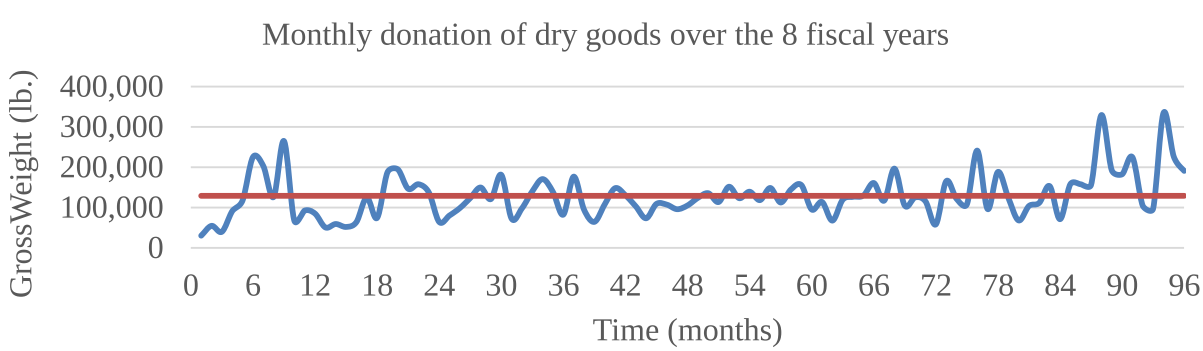

Figure 10. Monthly donation of dry goods for all fiscal years to Durham County.

4.3.1.2 Transfer-ins

The FBCENC branches transfer supplies among themselves. The transferring branch gives out items (transfer-out) to the receiving branch (transfer-in). Figure 11 shows a time-series graph of the monthly transfer-in of dry goods to the Durham branch over the eight fiscal years with monthly sample average transfer-in of approximately 289,000 lb. The minimum and the maximum transfer-in received are approximately 31,000 lb. and 646,000 lb. respectively.

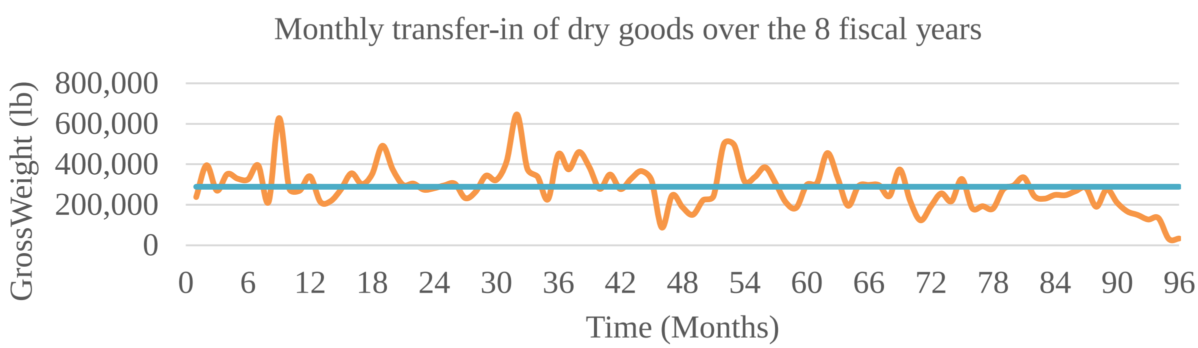

Figure 11. Monthly transfer-in of dry goods for all fiscal years to Durham County.

4.3.1.3 Available inventory

Available inventory represents the supplies that a branch (Durham in this case) uses to satisfy the demands of aid recipients in the various counties. These supplies consist of donations and transfer-in data less any transfer-out. Figure 12 shows a time-series graph of the monthly average available inventory of dry goods over the eight fiscal years with monthly average available inventory of approximately 418,000 lb. The minimum and the maximum transfer-in received are 195,000 lb. and 893,000 lb. respectively.

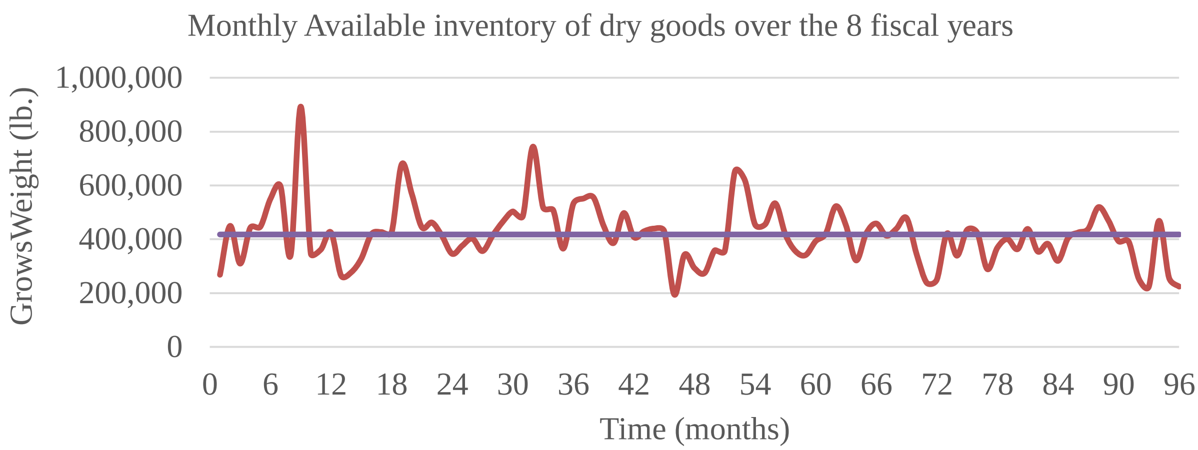

Figure 12. Monthly available inventory of dry goods for all fiscal years in Durham County.

4.3.1.4 Estimation of county's monthly demand

The data from FBCENC does not have records for the demand for each county based on the people in poverty. One could hypothesize that there is currently no standard mechanism in place to ensure that aid recipients request ahead of time the quantity of food they need. Thus, it is difficult for the FBCENC to accurately estimate the actual quantity of food each county might need. However, it is realistic to assume that there exists a correlation between the demand of a county and the poverty population of that county (Wight et al., 2014). Subsequently, the demand by county can be fairly estimated using the poverty population and a given pounds per person in poverty (PPIP) over a 12-month period, which is currently set at 75 pounds by the Feeding America. Table 6 shows the number of people living in poverty in each county and what their monthly projected demand should be in order to meet the 75 PPIP criterion over a 12-month period. It is over a 12-month period because the food bank would like to know how much food it distributes over an entire fiscal year. Equation (17) shows how the projected monthly demand is calculated for each county with poverty population, P_c.

$$Projected\ monthly\ demand = \frac{(P_c \times 75)}{12} \tag{17}$$

Table 6

Counties poverty populations and monthly projected demands

County	Poverty population	Projected monthly demand
Chatham	8,028	50,175
Durham	36,504	228,150
Granville	5,770	36,063
Orange	16,475	102,969
Person	5,829	36,431
Vance	10,859	67,869

4.3.1.5 Distribution

Figure 13 shows the average yearly distribution of dry goods by the Durham branch to the counties over the eight fiscal years. The distribution records are very important in this research because they provide information on the amount of food distributed over a 12 months period to aid recipients through various distribution programs. In addition, the PPIP calculation considers the history of previous distributions. Figure 13 shows the average yearly distribution of dry goods by Durham branch to the counties it serves.

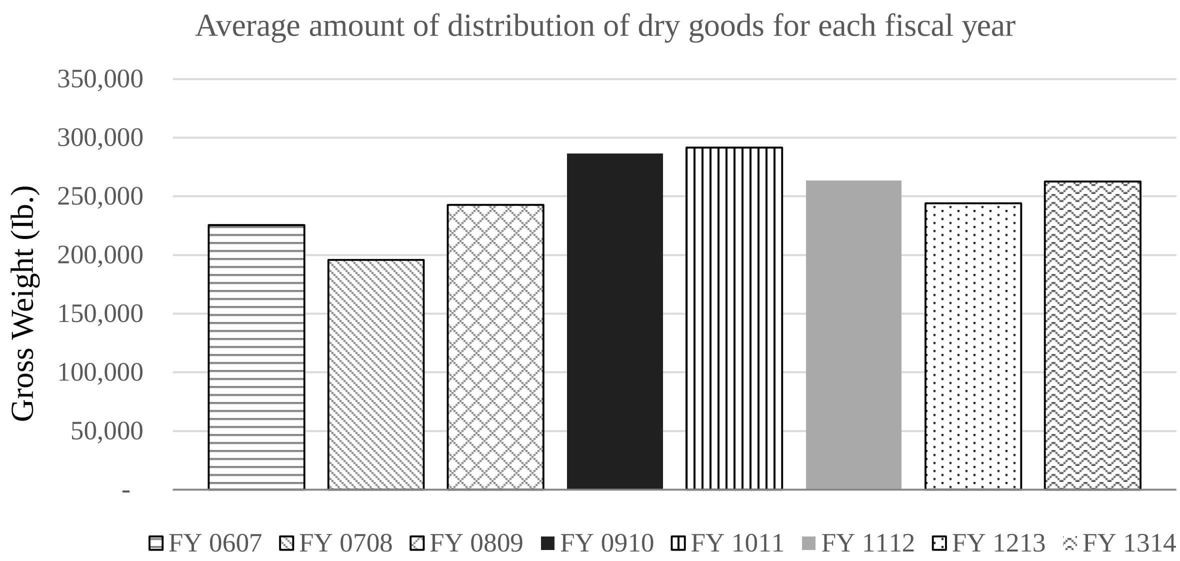

Figure 13. Average amount of distribution of dry goods for each fiscal year.

Figure 14 shows the average monthly distribution to counties over the eight fiscal years.

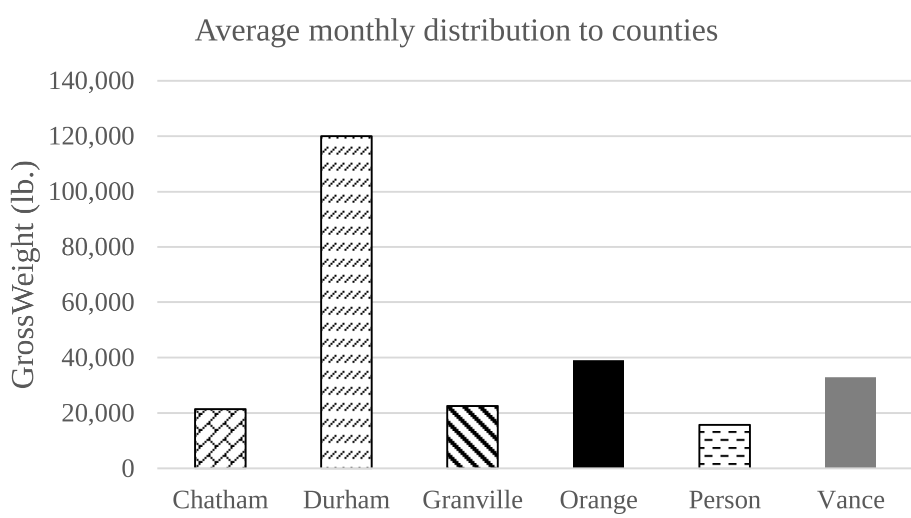

Figure 14 Average monthly distributions of dry goods to counties for all eight fiscal years.

This indicates on average how much is distributed to the counties on monthly bases.

4.4. Data Transformation

4.4.1 Discretization

The available inventory, donations and transfer-in datasets could assume any value between their minimum and maximum values. Thus, they could be considered to be continuous data. Consequently, these datasets need to be discretized in order to use them with the Discrete Time, Discrete State (DTDS) Markov Decision Process model. To do this, the percentage deviation of each monthly record from the average for each dataset is computed over the eight fiscal years. This transforms the original data from pounds into percentages above or below the mean for a given dataset. The binning technique, which was described in Chapter 3 is then used to group the transformed data into bins of equal width. The bin width used in this thesis is 10% for each dataset. The bins are associated with distinct discrete values using a one-to-one mapping as described in the Chapter 3. The sections below show the discretized values for available inventory, donation and transfer-in datasets.

4.4.1.1 Discretized available inventory

Figure 15 shows the mean percentage deviations for the available inventory dataset.

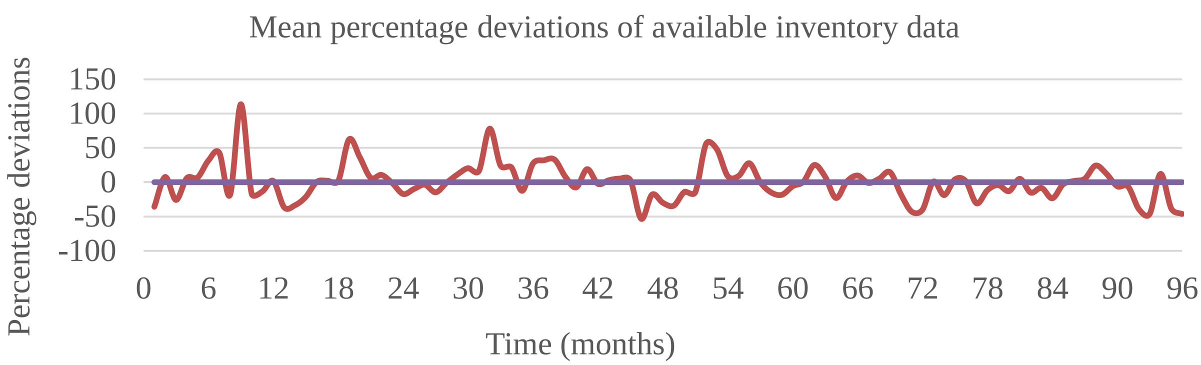

Figure 15. Percentages deviations from the available inventory mean for all eight fiscal years.

From figure 15 the minimum and maximum values are approximately -53% and 114% respectively.

Table 7 shows the mean percentage values grouped into bins with their discretized equivalents and the number of data points in each bin for the available inventory dataset.

Table 7

Discretization of available inventory data

Bin Range	Discretized form	Frequency
< -50	1	1
-50 to -40	2	4
-40 to -30	3	7
-30 to -20	4	5
-20 to -10	5	16
-10 to 0	6	13
0 to 10	7	26
10 to 20	8	7
20 to 30	9	7
30 to 40	10	4
40 to 50	11	2
50 to 60	12	1
60 to 70	13	1
70 to 80	14	1
80 to 90	15	0
> 90	16	1

The discretized values are used to represent the discrete space for the discrete Markov model. Once the percentage deviation is calculated for any given available inventory, the result is mapped into one of the bins and the discretized form is obtained as the state of the system.

4.4.1.2 Discretized donations

Figure 16 shows the mean percentage deviations calculated for the donations dataset.

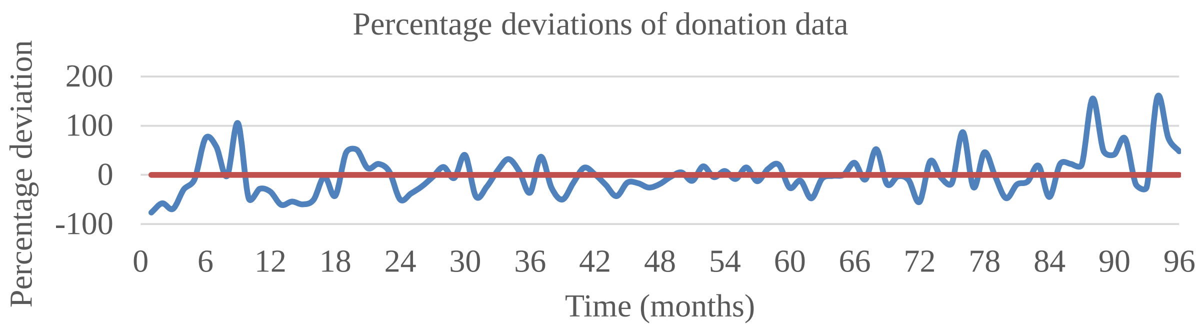

Figure 16. Percentages deviations from the donation mean over the eight fiscal years.

In Figure 16 the minimum and maximum values are approximately -77% and 159% respectively.

Table 8 shows the mean percentage deviations grouped into bins with their discretized equivalents and the number of data points in each bin for the donation dataset.

Table 8

Discretization of donations data

Bin Range	Discretized form	Frequency
< -70	1	1
-70 to -60	2	2
-60 to -50	3	8
-50 to -40	4	8
-40 to -30	5	3
-30 to -20	6	11
-20 to -10	7	14
-10 to 0	8	15
0 to 10	9	8
10 to 20	10	8
20 to 30	11	8
30 to 40	12	2
40 to 50	13	6
50 to 60	14	3
60 to 70	15	0
70 to 80	16	3
80 to 90	17	1
> 90	18	3

The median value of each bin range is used to calculate the incoming donation with exception of the extreme values. For the extreme bins, the actual values are used.

4.4.1.3 Discretized transfer-in

Figure 17 shows the mean percentage deviations calculated for the transfer-in data. From Figure 17 the minimum and maximum values are approximately -89% and 124% respectively.

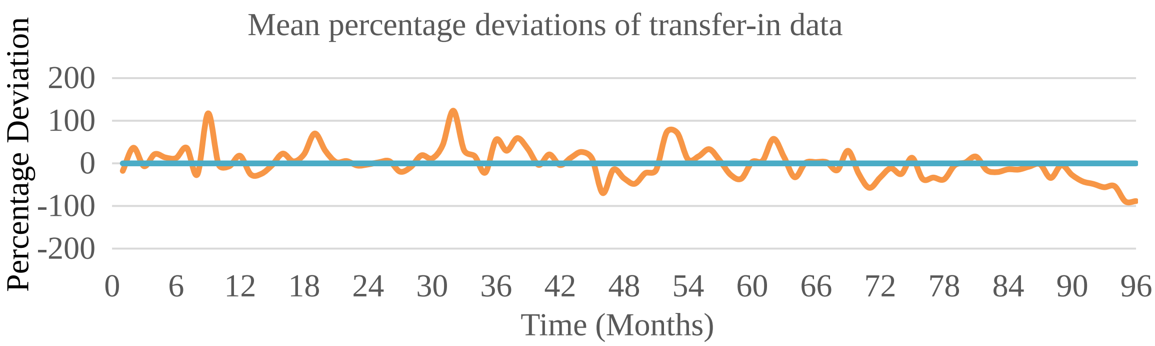

Figure 17. Percentages deviations from the mean of transfer-in dataset over the 8 fiscal years.

Table 9 shows the percentage mean deviations grouped into bins with their discretized equivalents and the number of data points in each bin.

Table 9

Discretization of transfer-in data

Bin Range	Discretized form	Frequency
< -80	1	2
-80 to -70	2	0
-70 to -60	3	1
-60 to -50	4	3
-50 to -40	5	3
-40 to -30	6	8
-30 to -20	7	10
-20 to -10	8	9
-10 to 0	9	13
0 to 10	10	13
10 to 20	11	12
20 to 30	12	8
30 to 40	13	5

Table 9

Cont.

40 to 50	14	1
50 to 60	15	3
60 to 70	16	0
70 to 80	17	3
80 to 90	18	0
> 90	19	2

The median value of each bin range is used to calculate the incoming transfer-in with exception of the extreme values. For the extreme bins, the actual values are used.

4.5 Probability Distributions

In probability theory, the Gaussian distribution is a continuous probability distribution that can be used to estimate the probability that any real observation will fall between any two real limits. Gaussian distributions are extremely important in statistics and are often used in the natural and social sciences for real-valued random variables. The Gaussian distribution is immensely useful because of the central limit theorem (CLT). The CLT states that, "the arithmetic mean of a sufficiently large number of iterates of independent random variables, each with a well-defined expected value and variance, will be normally distributed".

Equation (18) shows a Gaussian probability distribution function of a random variable x, having mean of μ and standard deviation of σ.

$$p(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \tag{18}$$

The normal probability distribution is also often denoted by $\mathcal{N}(\mu,\ \sigma^2)$. Thus when a random variable X is distributed normally with mean μ and variance σ^2, one can write it as $X\sim\mathcal{N}(\mu,\ \sigma^2)$.

The corresponding cumulative density function is given by equation (19).

$$\Phi(x,\mu,\sigma) = \frac{1}{2}\left[1 + \frac{\mathbf{erf}(x-\mu)}{\sigma\sqrt{2}}\right] \tag{19}$$

The $\boldsymbol{erf}(\bullet)$ is an error function term, which is given as: $\mathbf{erf}(x) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x} e^{-t^2}\,dt$.

With the cumulative probability function given, the probability that a random x will fall between x_1 and x_2 can be calculated using equation (20).

$$p(x_1 < x < x_2, \mu, \sigma) = \Phi(x_2, \mu, \sigma) - \Phi(x_1, \mu, \sigma) \tag{20}$$

The donations and transfers are the two the events that cause the available inventory to transition from one state to another state. These events have probability distributions associated with them. Using advanced statistical tools such JMP (by Statistical Analysis System Incorporation) or Matlab (by Mathworks Incorporation) one can model a probability distribution that best fits a given dataset. Using these tools, both the donation and transfer-in datasets follow Gaussian probability distributions with their respective mean and standard deviations as shown in Figure 18a and 18b respectively.

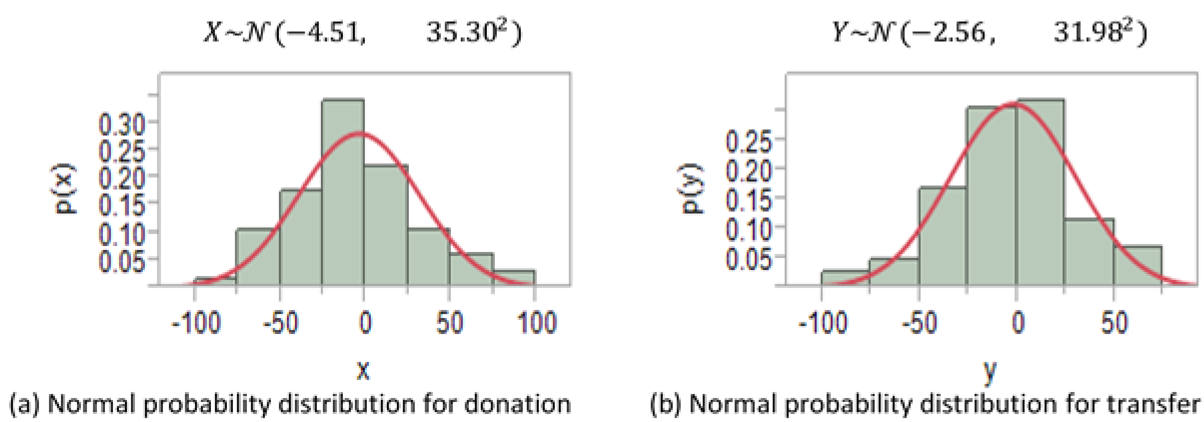

Figure 18. Normal probability distributions for the percentage deviations of donation and the transfer from their respective average values.

It should be noted from Figure 18 that the transformed donation dataset is normally distributed with mean, -4.51 and standard deviation, 35.30 while the transformed transfer-in dataset is normally distributed with mean, -2.56 and standard deviation, 31.98.

In order to confirm that the donation dataset is from a normal distribution, a goodness of fit test was conducted using the Shapiro-Wilk W Test and 5% significance level. From the results, the test statistic, W = 0.9812 and the p-value = 0.20. Since the p-value is greater than the significance value, it can be concluded that at the 5 % significant level, there is enough evidence to conclude that the donations data is from a normal distribution. Similarly, a goodness of fit test is also conducted to confirm that the transfer-in data is from a normal distribution a using the Shapiro-Wilk W Test. The significant level is 5%. From the results, the test statistic W = 0.9874 and the p-value = 0.51. Once again, since the p-value is greater than the significance value, it can be concluded that at the 5 % significant level, there is enough evidence to conclude that the transfer-in data is from a normal distribution

In what follows, we describe the various experiments that are run to test the developed discrete time, discrete space, Markov decision model.

4.6 Experimental Design

Various experimental designs are set up to analyze the optimal distribution policies for the FBCENC inventory system under investigation. The model is tested using varieties of inputs in order to answer the following research questions:

1. Should a fixed allocation policy be used at all time?
2. Can an allocation policy be defined generically based on different demand cases?
3. How does the large influx of supplies influence the allocation policy?

4.6.1 Base scenario

A base condition is established to gain insight into the optimal policy structure as well as the best reward. The base is tested with ideal monthly county demands. These demands are projected monthly demand that the county needs in order to meet the objective of distributing 75 pounds of food per person over a 12-month period. Thus, the results from this specific base scenario form the gold-standard to which all other scenarios are compared against. Table 10 shows the projected monthly demand for each county.

Table 10

County Poverty population and projected demand

County	Poverty population	Projected monthly demand (lb.)
Chatham	8,028	50,175
Durham	36,504	228,150
Granville	5,770	36,063
Orange	16,475	102,969
Person	5,829	36,431
Vance	10,859	67,869

These are ideal monthly demands that the food bank has to distribute to the counties in order to meet the 75 PPIP target over a 12-month period for each county.

Table 11 shows the approximated average monthly quantity of each historical dataset that is used in the model.

Table 11

Average monthly value of each historical dataset is used in the model

Dataset	Average value (Ib.)
Available inventory	418,000
Donations	129,000
Transfer-in	289,000

These values are essential as the inputs of the various scenarios that would be seen in this thesis are calculated relative to these values. More specifically, any given actual value is calculated as a percentage relative to the respective average values.

To compute the PPIP, the previous distribution to each county over eleven months needs to be known. These values are obtained by summing the monthly average distributions to each county over eight fiscal years. Table 12 shows these average monthly distributions for each county over all the fiscal years and their sum totals, which are represented as H_c.

Table 12

Average monthly distributions in pounds

Fiscal Month	Chatham	Durham	Granville	Orange	Person	Vance
08	18,272	103,118	20,164	30,193	12,931	28,544
09	17,691	100,273	22,762	34,038	12,272	27,282
10	19,102	115,151	22,520	39,918	15,931	29,447
11	21,733	124,864	21,865	44,370	13,941	34,106
12	26,213	126,460	21,409	39,567	13,681	33,326
01	24,397	139,616	27,086	40,689	19,677	33,172
02	23,176	138,804	26,345	44,339	18,494	35,238
03	24,618	142,957	25,074	47,917	16,525	37,396
04	21,712	125,203	22,977	43,834	18,450	36,730
05	20,385	109,634	22,195	33,427	17,729	29,980
06	20,136	110,619	22,143	29,984	17,469	30,206
Total (H_c)	**237,435**	**1,336,699**	**254,540**	**428,276**	**177,100**	**355,427**

Table 13 shows additional input parameters and their values used in the model.

Table 13

Summary of input parameters and their values

Parameter Name	Notation	Value
State space containing all possible states	V	{1, 2, .., 16}
Number of counties	C	6
Target PPIP	$PPIP_t$	75

Table 13

Cont.

Percentage of v allocated to county c for fixed policy	f_c	$1/C$
A set of allocation decisions	A	$\{1,2,3,4\}$
Time horizon (months)	τ	12

These parameter definitions and representations are general and extend to additional scenarios in the following experiments as well.

4.6.2 Sensitivity analyses

The model is built based on limited historical data from eight fiscal years, which may or may not be sufficient to accurately and robustly represent how the model would respond to unseen data. Since the actual county demands are unknown, the county demands become the major random variable that has great impact on the model. Consequently, a sensitivity analyses are performed to observe the optimal policy structure as a function of the variations in the county demands by varying the demand for each county relative to the projected monthly county demands. Also, variations in the donation and transfer-in are investigated. These variations are formulated in the next section.

4.6.2.1 Experiment 1: Evaluates the effect of changes in county demand

A county's demand is assumed to be proportional to the county's poverty population. However, the poverty population changes since families and individuals enter or leave this population for reasons such as relocation, job loss or new employment. To illustrate this changing demand as an outcome of the fluctuating poverty populations, different cases are developed with percentage variations as shown in Table 14.

Table 14 shows the various counties and their percentage variations relative to the base poverty population and projected monthly demands.

Table 14

Projected county demands

County	**Poverty population**	**Projected monthly demand (lb.)**	**Percentage Variations (%) in the poverty population and projected monthly demand**
Chatham	8,028	50,175	{-50,-40,-30,-20,-10,0,10,20,30,40,50,60,70,80,90,100}
Durham	36,504	228,150	{-50,-40,-30,-20,-10,0,10,20,30,40,50,60,70,80,90,100}
Granville	5,770	36,063	{-50,-40,-30,-20,-10,0,10,20,30,40,50,60,70,80,90,100}
Orange	16,475	102,969	{-50,-40,-30,-20,-10,0,10,20,30,40,50,60,70,80,90,100}
Person	5,829	36,431	{-50,-40,-30,-20,-10,0,10,20,30,40,50,60,70,80,90,100}
Vance	10,859	67,869	{-50,-40,-30,-20,-10,0,10,20,30,40,50,60,70,80,90,100}

For this experiment, it is assumed that each county has the same percentage increase or decrease relative to the reference. Thus, the experiment runs 16 different demand cases by adjusting each poverty population and projected monthly demand by the indicated percentages as shown on Table 14 for each county from -50% to 100% in increment of 10%. For each case, the optimal policy structure, and the expected quantity of food by which each county is underserved are compared to the base scenario (i.e. the results for the ideal projected monthly demands). In other words, the results corresponding to the combination, [0%, 0%, 0%, 0%, 0%, 0%], which represents, the percentage variation of the demand for Chatham, Durham, Granville, Orange, Person and Vance respectively.

4.6.2.2 Experiment 2: Evaluates the effect of changing supply

This experiment considers both the donation and transfer-in to fluctuate by the following percentages around their respective values and standard deviation as shown on Table 15.

Table 15

Percentage values of donation and transfer-in for changing supply

Parameter	**Value (Ib.)**	**Percentage Variation** (%)
Donation mean	129,000	{-50,-40,-30,-20,-10,0,10,20,30,40,50}
Donation Standard Deviation	35.30	{-50,-40,-30,-20,-10,0,10,20,30,40,50}
Transfer-in mean	289,000	{-50,-40,-30,-20,-10,0,10,20,30,40,50}
Transfer-in Standard Deviation	31.98	{-50,-40,-30,-20,-10,0,10,20,30,40,50}

There are 4 variables that are changing in this experiment as shown on Table 15; donation sample mean, donation standard deviation, transfer-in sample mean and transfer-in standard deviation. All these variables are considered to vary by -50% to 50% in increment of 10%. It should be noted that each of the 4 variables is varied one at a time, thus as one variable changes from -50% to 50% whiles the other three are kept at 0%.

4.6.2.3 Experiment 3: Evaluates the effect of non-stationary demand

The discrete state, discrete time MDP model formulated in this research assumes demand to be stationary over the entire 12 months period. This experiment examines the situation where the county demands are changing every 6 months. Once again, the monthly projected county demands are used as the reference for this sensitivity analysis. To start with, each county demand assumes the projected demand and remains unchanged for the first six months and then fluctuates for the next six months by a specified percentage as shown in Table 16.

Table 16 shows the various counties and their percentage variations for non-stationary demand cases.

Table 16

Percentage adjustments of county demands for non-stationary demand cases

Case	**Demand Growth (%)**	**Monthly County Demand (lb.)**					
		Chatham	**Durham**	**Granville**	**Orange**	**Person**	**Vance**
1	-50	25,088	114,075	18,032	51,485	18,216	33,935
2	-40	30,105	136,890	21,638	61,781	21,859	40,721
3	-30	35,123	159,705	25,244	72,078	25,502	47,508
4	-20	40,140	182,520	28,850	82,375	29,145	54,295
5	-10	45,158	205,335	32,457	92,672	32,788	61,082
6	0	50,175	228,150	36,063	102,969	36,431	67,869
7	10	55,193	250,965	39,669	113,266	40,074	74,656
8	20	60,210	273,780	43,276	123,563	43,717	81,443
9	30	65,228	296,595	46,882	133,860	47,360	88,230
10	40	70,245	319,410	50,488	144,157	51,003	95,017
11	50	75,263	342,225	54,095	154,454	54,647	101,804
12	60	80,280	365,040	57,701	164,750	58,290	108,590
13	70	85,298	387,855	61,307	175,047	61,933	115,377
14	80	90,315	410,670	64,913	185,344	65,576	122,164
15	90	95,333	433,485	68,520	195,641	69,219	128,951
16	100	100,350	456,300	72,126	205,938	72,862	135,738

For this analysis also, it is assumed that each county has the same percentage increase or decrease relative to the reference. Specifically, the percentage adjustments for each county's demand varies from -50% to 100% increment of 10% for all counties. This generated a total of 16 demand cases. In each case, the system begins with 0% demand growth for the first six months and takes on different demand growth for the next six months.

CHAPTER 5

Results and Discussion

5.1 Overview

5.1.1 Optimal policies

This chapter provides and discusses the results of all the three experiments that were described in Chapter 4. The optimal policy is displayed as a matrix, the rows are the pseudo states of the available inventory and the columns represent the time remaining until the end of the time horizon. Figure 19 illustrates a sample policy structure.

			Best Policy									
	Time period (month)											
States	1	2	3	4	5	6	7	8	9	10	11	12
1	3	3	3	3	3	3	3	3	3	3	3	3
2	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	3	3	3	3	3
4	3	3	3	3	3	3	3	3	3	3	3	3
5	3	3	3	3	3	3	3	3	3	3	3	3
6	3	3	3	3	3	3	3	3	3	3	3	3
7	3	3	3	3	3	3	3	3	3	3	3	3
8	3	3	3	3	3	3	3	3	3	3	3	3
9	123	123	123	123	123	123	123	123	123	123	123	123
10	123	123	123	123	123	123	123	123	123	123	123	123
11	123	123	123	123	123	123	123	123	123	123	123	123
12	123	123	123	123	123	123	123	123	123	123	123	123
13	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234
14	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234
15	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234
16	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234

Figure 19. Illustration of sample policy structure.

In Figure 19, state 1 and state 16 are the minimum and maximum available inventory state respectively. The maximum time period is 12 months, which indicates the end of the time horizon. There are 16 states as shown on Figure 19. The set of numbers shown under each time period represents a policy, each one of which corresponds to a given state that tells a decision-maker, an allocation that should be adopted at that state. For instance, under time period 1, one sees {3, 3, 3, 3, 3, 3, 3, 3, 123, 123, 123, 123, 1234, 1234, 1234, 1234} corresponding to state 1 through 16 respectively. In this case from state 1 through state 8, allocation rule, 3 is to be adopted by the decision-maker. There are only four allocation rules, which are represented as 1, 2, 3 and 4 respectively. In a situation where two or more allocation rules could be adopted, these values are concatenated together. For instance, from state 9 through state 12, one sees allocation rule 123. This means the decision-maker could either use allocation rule 1, or 2 or 3. Similarly, from state 13 through states 16, one sees allocation rule 1234, which means that allocation rule, 1 or 2 or 3 or 4 could be used as all give the same optimal solution.

It should be noted that there are 15 possible combinations of allocation rules (1, 2, 3, 4, 12, 13, 14, 23, 24, 34, 123, 124, 134, 234, 1234) that may be optimal for a given available inventory state at a specific time period.

5.1.2 Equity

The reward associated with the optimal policy is equity. This is also displayed as a matrix; the rows are the pseudo states of the available inventory and the columns represent the time remaining until the end of the time horizon. Perfect equity equals 1 and this is achieved when each person in poverty in the various counties receives equal share of the pounds of food distributed. Figure 20 illustrates a sample reward (equity) structure. In Figure 20, the supplies are equitably distributed for each state at each time horizon hence the equity at the end of the time horizon is 12 for all states. This is an ideal scenario, which is what the food bank wants.

	Equity											
	Time period (month)											
States	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	12	11	10	9	8	7	6	5	4	3	2	1
2	12	11	10	9	8	7	6	5	4	3	2	1
3	12	11	10	9	8	7	6	5	4	3	2	1
4	12	11	10	9	8	7	6	5	4	3	2	1
5	12	11	10	9	8	7	6	5	4	3	2	1
6	12	11	10	9	8	7	6	5	4	3	2	1
7	12	11	10	9	8	7	6	5	4	3	2	1
8	12	11	10	9	8	7	6	5	4	3	2	1
9	12	11	10	9	8	7	6	5	4	3	2	1
10	12	11	10	9	8	7	6	5	4	3	2	1
11	12	11	10	9	8	7	6	5	4	3	2	1
12	12	11	10	9	8	7	6	5	4	3	2	1
13	12	11	10	9	8	7	6	5	4	3	2	1
14	12	11	10	9	8	7	6	5	4	3	2	1
15	12	11	10	9	8	7	6	5	4	3	2	1
16	12	11	10	9	8	7	6	5	4	3	2	1

Figure 20. Illustration of sample reward structure.

5.1.3 Unsatisfied demand per person

Based on the criterion set by Feeding America, the unsatisfied demand is the amount of additional supplies needed by each person in poverty in each county in order to meet the target PPIP of 75 pounds of food. The amount of unsatisfied demand is calculated for each county using the optimal policy. Figure 21 shows the unsatisfied demand for a given county. A zero value at any given state under any time period is an indication that the person in poverty received at least 75 pounds of supplies and hence needs no extra supply. On the contrary, non-zero value indicates the extra amount of supply that each person in poverty needs in order to meet the 75 pounds target.

		Unsatisfied Demand										
	Time period (month)											
States	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	23	21	19	17	15	14	12	10	8	6	5	3
2	22	21	19	17	15	13	12	10	8	6	4	3
3	22	20	18	16	15	13	11	9	7	6	4	2
4	21	20	18	16	14	12	11	9	7	5	3	2
5	21	19	17	15	14	12	10	8	6	5	3	1
6	20	19	17	15	13	11	10	8	6	4	2	1
7	20	18	16	15	13	11	9	7	6	4	2	0
8	20	18	16	15	13	11	9	7	5	4	2	0
9	20	18	16	14	13	11	9	7	5	4	2	0
10	20	18	16	14	13	11	9	7	5	4	2	0
11	20	18	16	14	13	11	9	7	5	4	2	0
12	20	18	16	14	13	11	9	7	5	4	2	0
13	20	18	16	14	12	11	9	7	5	3	1	0
14	19	17	16	14	12	10	8	7	5	3	1	0
15	19	17	15	13	12	10	8	6	4	3	1	0
16	19	17	15	13	12	10	8	6	4	2	1	0

Figure 21. Illustration of sample unsatisfied demand for Chatham.

5.1.4 Number of counties underserved

After calculating the unsatisfied demand, further analysis is performed to identify the number of counties that are underserved as a result of the state of the available inventory and all the possible donations that can occur. The underserved counties are those counties whose PPIPs fall below the target PPIP of 75 pounds. There are 16 states that the available inventory could assume. For each state, there are 18 different ways donation could come in and this determines the amount of supply each county could receive. Consequently, the number of counties underserved depending on the available inventory and the donation that comes in prior to the distribution of the supply to the counties is calculated as an expected value.

5.2 Base scenario

The main variables that are considered in the experimental analyses are the donation, transfer-in and the demand. For the base scenario, these parameters remain within their expected values, which are determined based on historical data of eight fiscal years from the FBCENC. These values are shown in Table 17 and 18 below.

Table 17

Sample mean and the standard deviation values for the donation and transfer-in

Dataset	Sample Mean (lb.)	Standard Deviation
Donation	129,000	35.30
Transfer-in	289,000	31.98
Available Inventory	418,000	N/A

The standard deviation of the available inventory is not included as the available inventory is not modeled as a normal distribution. In order words, the available inventory is not an event that causes a transition. Donation and transfer-in cause the available inventory to change states.

Table 18

County demands

County	Projected monthly demand (lb.)
Chatham	50,175
Durham	228,150
Granville	36,063
Orange	102,969
Person	36,431
Vance	67,869
Total	**521,657**

Once again the values shown on Table 18 are ideal county demands that would guarantee a PPIP of 75 pounds for each county at the end of the 12-month period.

5.2.1 Optimal policies for base scenario

Figure 22, shows the optimal policy for the base scenario.

Base Scenario		Best Policy											
		Time period (month)											
States		1	2	3	4	5	6	7	8	9	10	11	12
< -50%	1	3	3	3	3	3	3	3	3	3	3	3	3
-45%	2	3	3	3	3	3	3	3	3	3	3	3	3
-35%	3	3	3	3	3	3	3	3	3	3	3	3	3
-25%	4	3	3	3	3	3	3	3	3	3	3	3	3
-15%	5	3	3	3	3	3	3	3	3	3	3	3	3
-5%	6	3	3	3	3	3	3	3	3	3	3	3	3
5%	7	3	3	3	3	3	3	3	3	3	3	3	3
15%	8	3	3	3	3	3	3	3	3	3	3	3	3
25%	9	123	123	123	123	123	123	123	123	123	123	123	123
35%	10	123	123	123	123	123	123	123	123	123	123	123	123
45%	11	123	123	123	123	123	123	123	123	123	123	123	123
55%	12	123	123	123	123	123	123	123	123	123	123	123	123
65%	13	123	123	123	123	123	123	123	123	123	123	123	123
75%	14	123	123	123	123	123	123	123	123	123	123	123	123
85%	15	123	123	123	123	123	123	123	123	123	123	123	123
> 90%	16	123	123	123	123	123	123	123	123	123	123	123	123

Figure 22. Optimal policy for base scenario.

From Figure 22, the optimal policy structure is stationary for the base scenario because irrespective of the time horizon, the policy structure is the same. The interpretation of the optimal policy structure is as follows:

1. From states 1 through to 8, irrespective of the time horizon, allocation rule 3 is optimal;
2. From states 9 through to 16, irrespective of the time horizon, allocation rules 1 or 2 or 3 are optimal;
3. Rule 3 is the best among all the rules since it is optimal for each time period irrespective of the state.

These results are realistic because the total demand for all the counties is approximately 521,700 lb. In state 1, the available inventory corresponds to 208,900 lb. of supplies (50% below the mean), and at state 8, the available inventory corresponds 480,470 lb. (15% above the mean).

These available inventory states are highly constrained because the inventories in these states are not sufficient to satisfy all the demands from the various counties. In addition, there are only some occurrences of donations that the total supply available for distribution is able to satisfy all the demands. Hence, allocation rule in which each county receives a portion of the supply based on the county's poverty population (i.e. rule 3) maximizes equity. Further analyses, indicate that, at state 9, the available inventory corresponds to approximately 552,250 lb. (25% above the mean) and that of state 16 corresponds to approximately 793,800 lb. (90% above the mean). Accordingly, from state 9 through state 16, there is enough supply to satisfy all the county demands irrespective of the incoming donation. Thus, allocation rule 1, 2 or 3 is optimal. In other words, whether the largest demand is served first; or the smallest demand is served first; or supply is distributed according to poverty population, equity is maximized.

It is observed that rule 3 is dominant in the optimal policy structure for each time period irrespective of the states. Accordingly, allocation rule 3 is the best amongst all the rules because rule 3 maximizes equity irrespective of the state of the available inventory and time period. Rules 1 and 2 are optimal only when there are sufficient supplies to satisfy all the demand for each county.

5.2.2 Equity for base scenario

Figure 23 shows the reward (equity) for the base scenario.

	Base Scenario	Equity											
		Time period (month)											
States		1	2	3	4	5	6	7	8	9	10	11	12
< -50%	1	12	11	10	9	8	7	6	5	4	3	2	1
-45%	2	12	11	10	9	8	7	6	5	4	3	2	1
-35%	3	12	11	10	9	8	7	6	5	4	3	2	1
-25%	4	12	11	10	9	8	7	6	5	4	3	2	1
-15%	5	12	11	10	9	8	7	6	5	4	3	2	1
-5%	6	12	11	10	9	8	7	6	5	4	3	2	1
5%	7	12	11	10	9	8	7	6	5	4	3	2	1
15%	8	12	11	10	9	8	7	6	5	4	3	2	1
25%	9	12	11	10	9	8	7	6	5	4	3	2	1
35%	10	12	11	10	9	8	7	6	5	4	3	2	1
45%	11	12	11	10	9	8	7	6	5	4	3	2	1
55%	12	12	11	10	9	8	7	6	5	4	3	2	1
65%	13	12	11	10	9	8	7	6	5	4	3	2	1
75%	14	12	11	10	9	8	7	6	5	4	3	2	1
85%	15	12	11	10	9	8	7	6	5	4	3	2	1
> 90	16	12	11	10	9	8	7	6	5	4	3	2	1

Figure 23. Equity for base scenario.

In Figure 23, the supply is equitably distributed in each state at each time horizon. This is seen in the last time horizon where equity is 12 for all states. This implies that using the optimal policy results in an equitable distribution of supplies irrespective of the state in which the available inventory is.

5.2.3 Unsatisfied demand for base scenario

The unsatisfied demand is calculated using the optimal policy for each state at each time period. Thus, for a state and time period, the rule that immerges as optimal is used to calculate what the unsatisfied demands for that state and time period are. Accordingly, based on the optimal policy structure irrespective of the time period, from states 1 through to state 8, allocation rule 3 is used to calculate the unsatisfied demand for each county. From states 9 through 16, rules 1, 2 or 3 are used to calculate the unsatisfied demand for each county. However, since rule 3 is optimal irrespective of the state and time period, the unsatisfied demand for rule 3 is displayed for each state at each time period for each county. Using rule 3, the unsatisfied demand for each state at each time period is the same for all the counties. Thus, only one result is displayed. From Figure 24, at the end of the time horizon for state 1 the unsatisfied demand is 21.81 pounds per person in poverty; in state 2 the unsatisfied demand is 21.61 pounds per person in poverty; and in state 3 the unsatisfied demand is 21.05 pounds per person in poverty. There is a decreasing trend in the unsatisfied demand moving from state 1 to 16 as shown in figure 24.

	Base Scenario	Unsatisfied Demand											
		Time period (month)											
States		1	2	3	4	5	6	7	8	9	10	11	12
< -50%	1	21.81	20.09	18.37	16.65	14.92	13.20	11.48	9.76	8.03	6.30	4.56	2.75
-45%	2	21.61	19.89	18.17	16.45	14.72	13.00	11.28	9.56	7.83	6.10	4.36	2.55
-35%	3	21.05	19.33	17.61	15.89	14.17	12.45	10.72	9.00	7.28	5.55	3.80	1.99
-25%	4	20.60	18.88	17.16	15.44	13.72	12.00	10.27	8.55	6.83	5.10	3.35	1.55
-15%	5	20.00	18.28	16.56	14.84	13.12	11.39	9.67	7.95	6.23	4.50	2.76	0.99
-5%	6	19.43	17.71	15.99	14.27	12.55	10.83	9.10	7.38	5.66	3.94	2.22	0.58
5%	7	18.68	16.96	15.23	13.51	11.79	10.07	8.35	6.63	4.91	3.21	1.55	0.16
15%	8	18.07	16.34	14.62	12.90	11.18	9.46	7.74	6.03	4.32	2.65	1.10	0.03
25%	9	17.46	15.74	14.02	12.30	10.58	8.86	7.15	5.44	3.76	2.15	0.75	0
35%	10	16.87	15.14	13.42	11.70	9.98	8.27	6.56	4.87	3.22	1.69	0.48	0
45%	11	16.25	14.53	12.81	11.09	9.37	7.66	5.96	4.29	2.70	1.28	0.28	0
55%	12	15.64	13.92	12.20	10.48	8.76	7.06	5.37	3.74	2.21	0.93	0.15	0
65%	13	15.03	13.31	11.59	9.87	8.16	6.47	4.80	3.21	1.77	0.65	0.07	0
75%	14	14.44	12.72	11.00	9.29	7.58	5.90	4.26	2.72	1.38	0.43	0.03	0
85%	15	13.88	12.16	10.44	8.73	7.03	5.36	3.75	2.28	1.06	0.27	0.01	0
> 90	16	13.59	11.87	10.15	8.44	6.75	5.08	3.50	2.06	0.91	0.21	0.01	0

Figure 24. Showing the counties' expected unsatisfied demands for base scenario.

At the end of the time horizon, the maximum expected unsatisfied demand is approximately 22 pounds per person in poverty, which is recorded in state 1 and the minimum expected unsatisfied demand is approximately 14 pounds per person in poverty, which is recorded in state 16.

5.2.4 Number of counties underserved for base scenario

Underserved counties are those counties whose PPIPs fall below the target PPIP of 75 pounds. The available inventory and the donation have great impacts on the number of counties that are underserved or well-served. As explained earlier, there are 16 states and for each of those states, there are 18 possible donations that can occur (refer to Chapter 4 section 4.4.1 for the details of the 18 possible donations). At each state (available inventory) and at specific donation, a minimum of 0 counties could be underserved and a maximum of all 6 counties could be underserved. This is directly attributable to the optimal policy structure. Accordingly, the optimal policy structure is used to estimate the expected number of counties that are underserved for each state. Figure 25 shows the expected number of counties underserved in each state at the end of the time horizon.

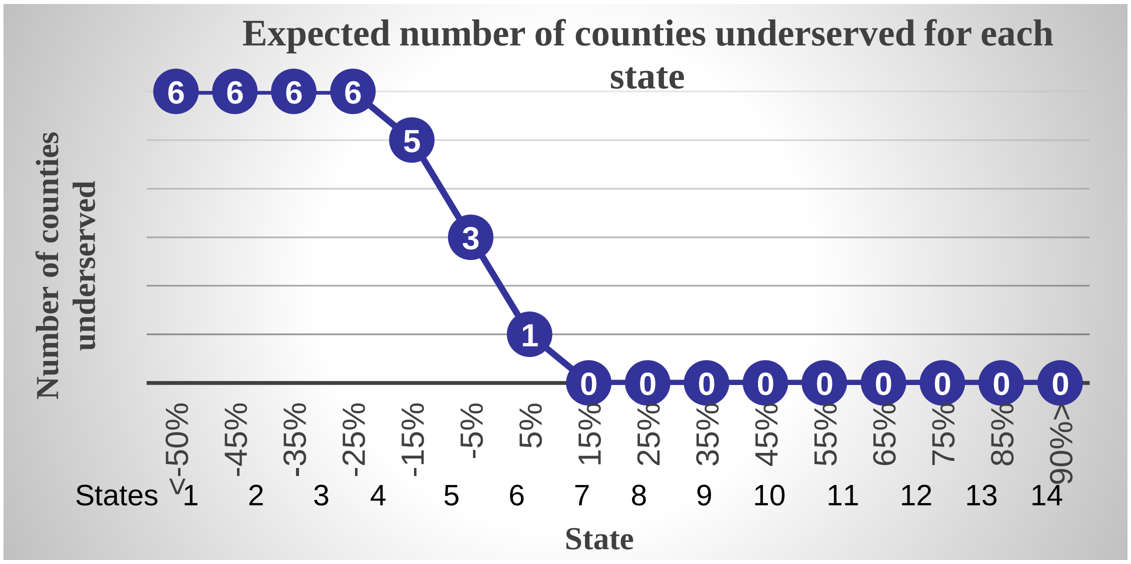

Figure 25. Illustrating the expected number of counties underserved for the base scenario.

In Figure 25, if the available inventory is -50% to 5% (states 1 to 7) relative to the sample mean at least one of the 6 counties is underserved. However, if the available inventory is kept at least 15% (from state 8) above the sample mean, no counties are underserved. In other words, all the counties would be well-served so far as the base scenario is concerned.

5.3 Sensitivity Analyses

The sensitivity analyses are performed to observe the changes in the optimal policy, unsatisfied demand and the number of counties underserved compared to that of the base scenario. In this case, the demands and the supply are varied to generate different cases to test the behavior of the model. Using the results from the base scenario as the gold-standard, the error (deviation) for each case is estimated using equation 21.

$$Error = Measured\ result - Base\ result \tag{21}$$

From equation (21), the error is the difference between a specific measured result of a specific case and that of the base scenario result.

It should be noted that the unsatisfied demand and the number of counties underserved at the end of the time horizon are investigated by comparing them to that of base scenario at the last time horizon. This is crucial as the food bank measures these parameters at the end of a 12-month period.

5.3.1 Effects of changing demand

5.3.1.1 Optimal policy for demand cases

This experiment generated 16 demand cases by varying the demand for each of the six counties from -50% to 100% in increment of 10%. Figure 26 shows how the optimal policy structure changes as a result of the variations in the county demands. For the optimal policy structure, only the constrained states are investigated since these are the states where the supply is not enough to satisfy all the demand.

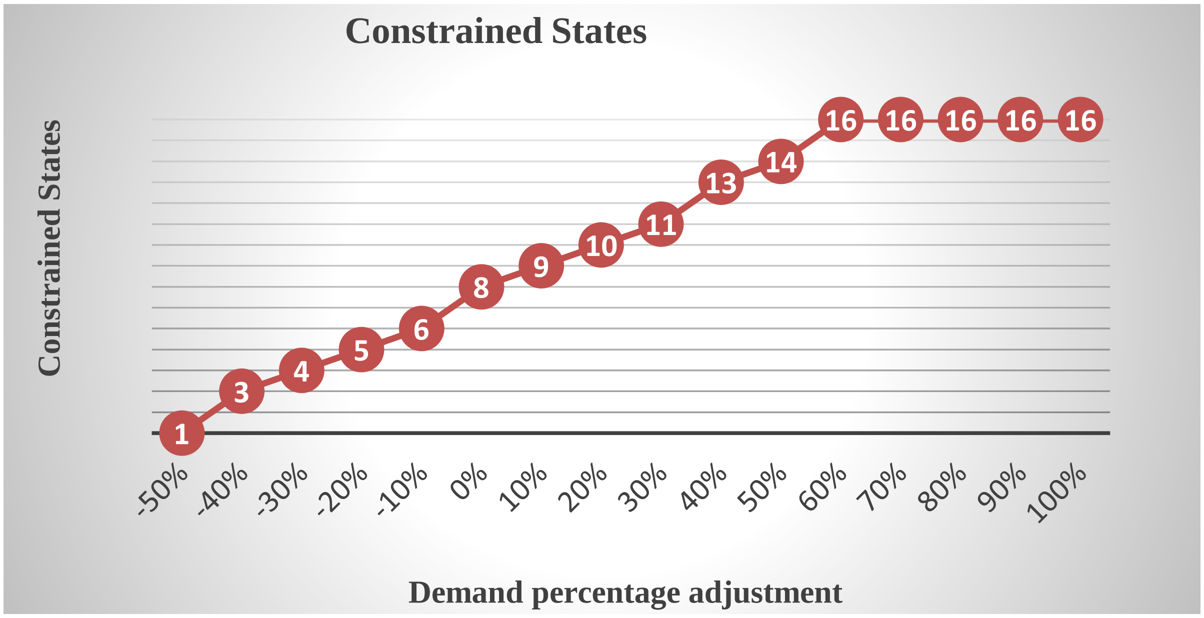

Figure 26. Optimal policy for demand cases.

Figure 26 is interpreted as follows: when the demand percentage change is -50% (50% below the projected demand) only available inventory state 1 is constrained. However, when the demand is 60% to 100% above the projected demand all the 16 available inventory states are constrained. In general, it can be observed that as the county demand increases from -50% to 100% the number of constrained available states also increases from 1 to 16.

5.3.1.2 Unsatisfied demand for all counties for demand cases

The unsatisfied demand at the end of the time horizon is calculated based on the optimal policy structure. Because rule 3 is the best amongst all the rules as seen in the optimal policy structure, rule 3 is used to calculate the unsatisfied demand. Using rule 3, it was observed that, the expected unsatisfied demand for each state at each time period is the same for all the counties hence, only one of such results is displayed to represent all the counties. The result for the unsatisfied demand is displayed for states 1, 8 and 16 in Figure 26.

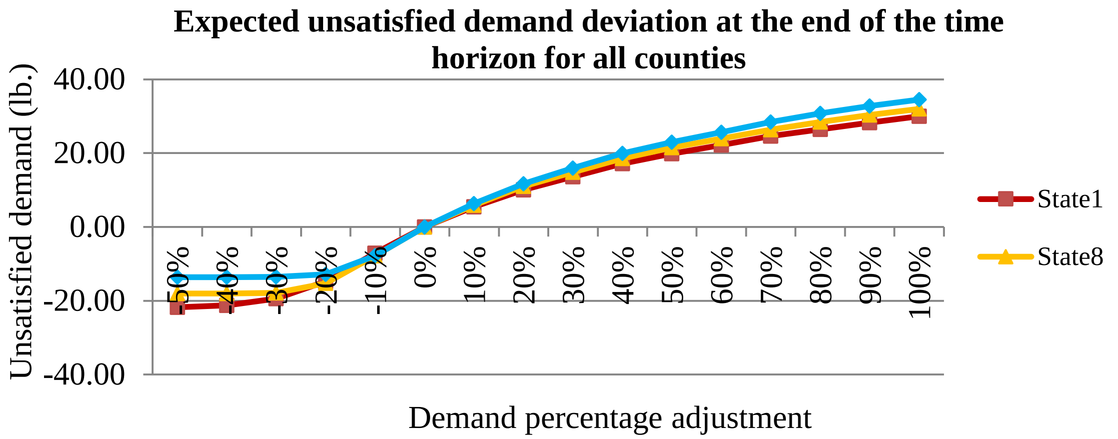

Figure 27. Expected unsatisfied demand for all demand cases.

From Figure 27, irrespective of the state, -50 to -10% demand percentage adjustment showed negative deviations from the base. Negative deviations indicate smaller unsatisfied demand compared to that of the base. However, as the demand percentage adjustments increased from 10% to 100% the deviations are positive. Positive deviations indicate unsatisfied demand compared to that of the base becomes larger. In general, it can be observed that the deviation in the unsatisfied demand at the end of the time horizon from the base increases monotonically as the demand percentage adjustment increases.

5.3.1.3 Number of counties underserved for demand cases

The result for the number of counties underserved is as shown in Figure 28.

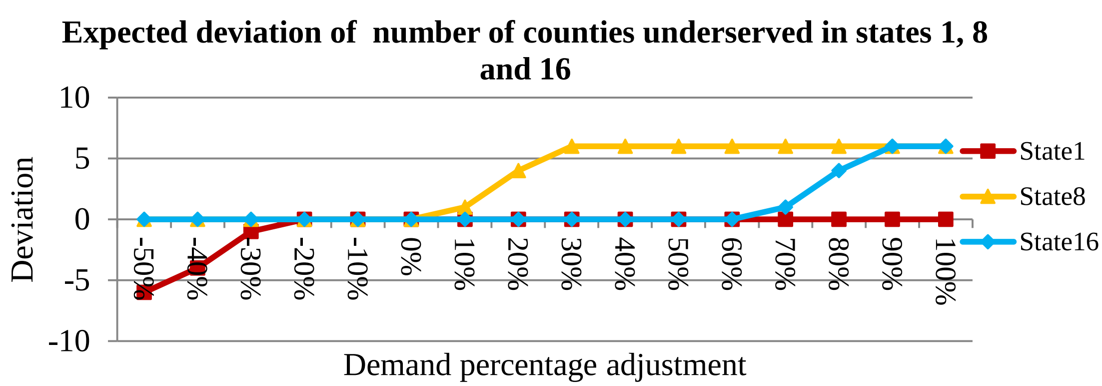

Figure 28. Expected deviation of the number of counties underserved in states 1, 8 and 16.

From Figure 28, -50 to -20% demand percentage adjustment showed negative deviations from that of the base for state 1. Negative deviations indicate smaller number of counties underserved compared to that of the base. However, as the demand percentage adjustments increased from -20% to 100% the deviations are zero for state 1. Zero deviations indicate no change in the number of counties underserved compared to that of the base.

In state 8, -50% to 0% demand percentage adjustment showed no deviations from the base. However, as the demand percentage adjustments increased from 0% to 100% the deviations are positive. Positive deviations indicate bigger number of counties underserved compared to the base.

In state 16, -50 to 60% demand percentage adjustments showed no deviations from the base. However, as the demand percentage adjustments increased from 60% to 100% the deviations are positive.

In general it can be observed that the deviation from the base for the number of counties underserved at the end of the time horizon increases monotonically as the demand percentage adjustment increases.

5.3.2 Effects of changing supply

There are 4 variables that are changing in this experiment; the donation sample mean, the donation standard deviation, the transfer-in sample mean and the transfer-in standard deviation. A total of 4 different scenario are generated based on the 4 variables. A scenario corresponds to the situation where one variable changes from -50% to 50% while the other 3 variables remain the same at 0% above their base values. The result of each of the 4 scenarios at the end of the time horizon is compared to that of the base scenario and the error analyzed.

5.3.2.1 Optimal policies for supply cases

The result for changes in the optimal policy structure for each of the four variables is shown below. For the optimal policy structure for each of the 4 scenarios only the constrained states are investigated since these are the states where the supply is not enough to satisfy all the demand. Figure 29 shows the scenario where the donation sample mean varies from -50% to 50% while all the other three parameters remain unchanged.

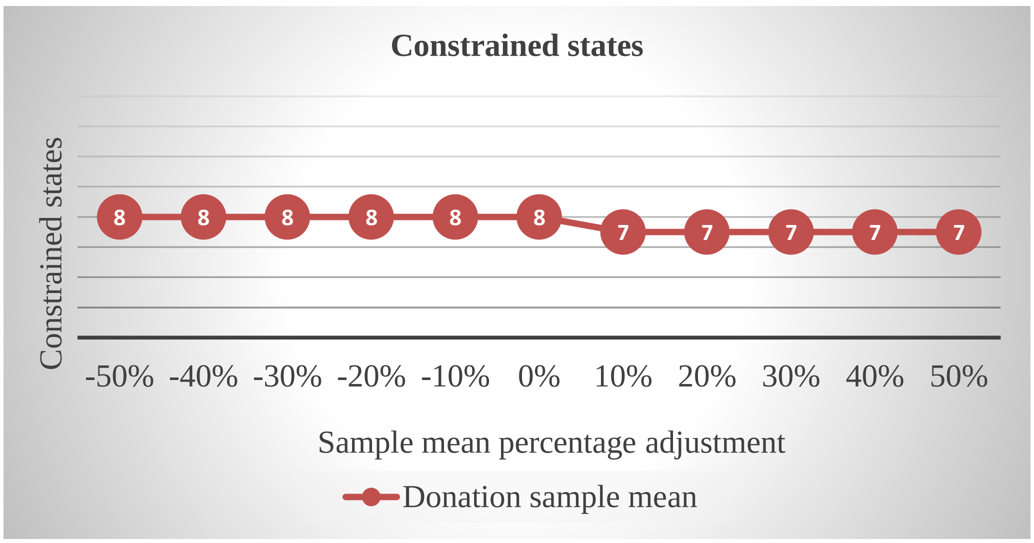

Figure 29. Optimal policy structure for scenario1.

Figure 29 is interpreted as follows: when the donation sample mean percentage adjustment increases -50% to 0%, 8 available inventory states are constrained. The 8 constrained states are state 1 through to state 8. However, when the donation sample mean increase from 0% to 50% only 7 available inventory states are constrained (states 1 through to 7).

Figure 30 shows the scenario where the donation standard deviation varies from -50% to 50% while all the other three parameters remain unchanged.

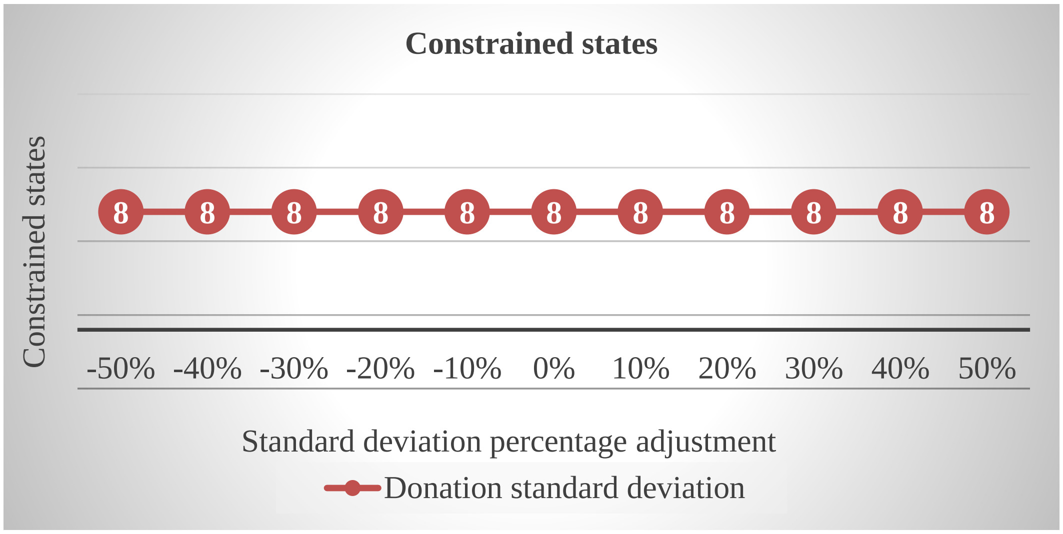

Figure 30. Optimal policy structure for scenario 2.

From Figure 30, when the donation standard deviation percentage adjustment increases from -50% to 100%, 8 available inventory states are constrained. The 8 constrained sates are state 1 through to state 8. It should be noted this policy structure is the same as that of the base scenario. This implies that when the standard deviation of the donation increases from -50% to 50%, the optimal policy does not change.

Figure 31 shows the scenario where the transfer-in sample mean varies from -50% to 50% while all the other three parameters remain unchanged.

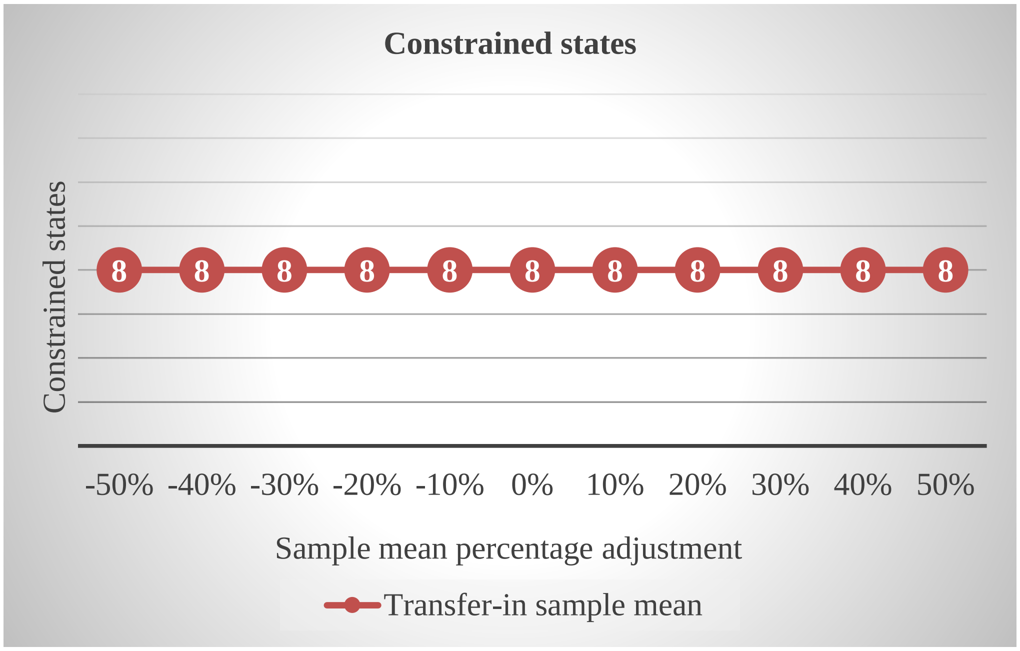

Figure 31. Optimal policy structure for scenario 3.

From Figure 31, when the transfer-in sample mean percentage adjustment increases from -50% to 100%, 8 available inventory states are constrained. The 8 constrained sates are state 1 through to state 8. This policy structure is also the same as that of the base scenario. This implies that when the transfer-in sample mean increases from -50% to 50%, the optimal policy does not change.

Figure 32 shows the scenario where the transfer-in standard deviation varies from -50% to 50% while all the other three parameters remain unchanged.

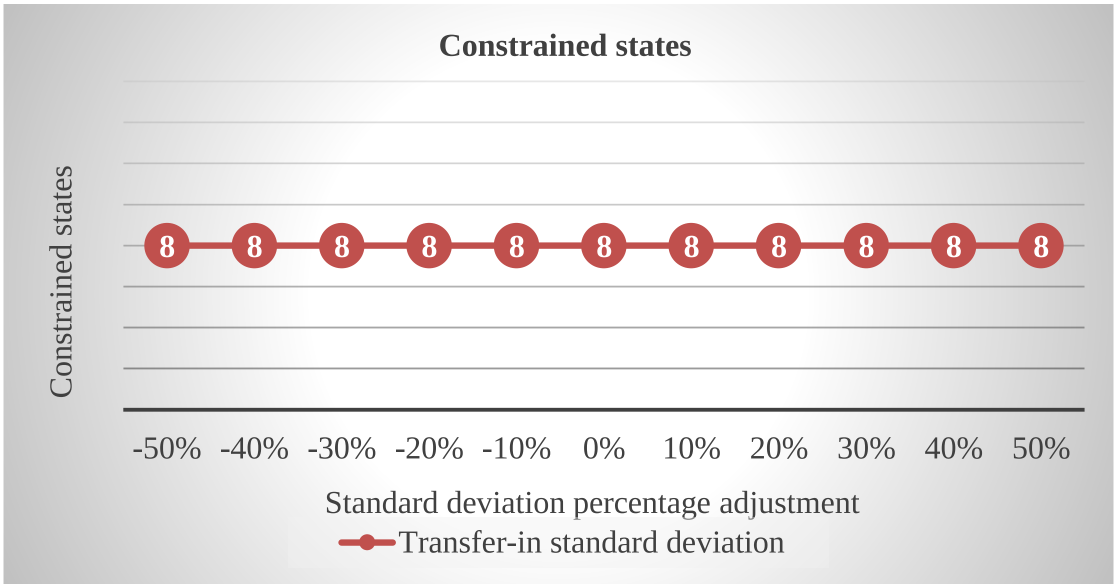

Figure 32. Optimal policy structure for scenario 4.

From Figure 32 when the transfer-in standard deviation percentage adjustment increases from -50% to 100%, 8 available inventory states are constrained. The 8 constrained sates are state 1 through to state 8. This policy structure is also the same as that of the base scenario. This implies that when the standard deviation of transfer-in increases from -50% to 50%, the optimal policy does not change.

In general, from the optimal policy structure of the 4 scenarios investigated under this experiment, only changes in the donation sample mean has an effect on the optimal policy structure.

5.3.2.2 Unsatisfied demand for supply cases

The result of unsatisfied demand for each state at the end of the time horizon for all the supply scenarios is calculated and compared to that of the base scenario. However, the results are displayed for only state 1 (lowest inventory level), state 8 (average inventory level) and state 16 (highest inventory level). The error is calculated to see how far the results of these scenarios deviate from that of the base scenario.

Figure 33 shows the unsatisfied demand for state 1 at the end of the time horizon for all the supply cases compared to that of the base scenario.

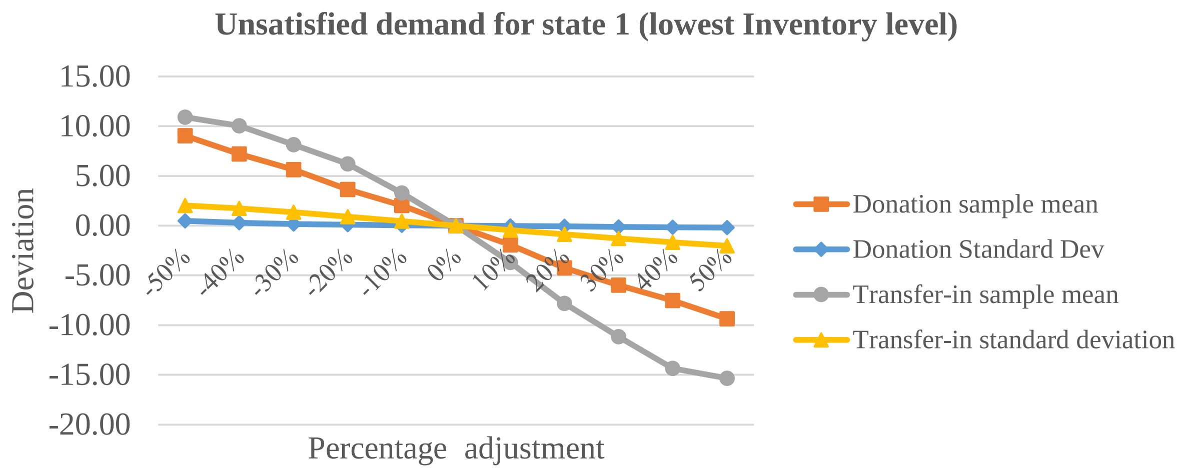

Figure 33. Deviation of unsatisfied demand for state 1.

From Figure 32, -50% to 0% percentage adjustment showed positive deviations from the base for all the 4 scenarios investigated. Positive deviations indicate higher unsatisfied demand compared to the base. However, as the supply percentage adjustments increased from 0% to 50% the deviations are negative. Negative deviations indicate lower unsatisfied demand compared to that of the base scenario.

Figure 34 shows the unsatisfied demand for state 8 at the end of the time horizon for all the supply cases compared to that of the base scenario.

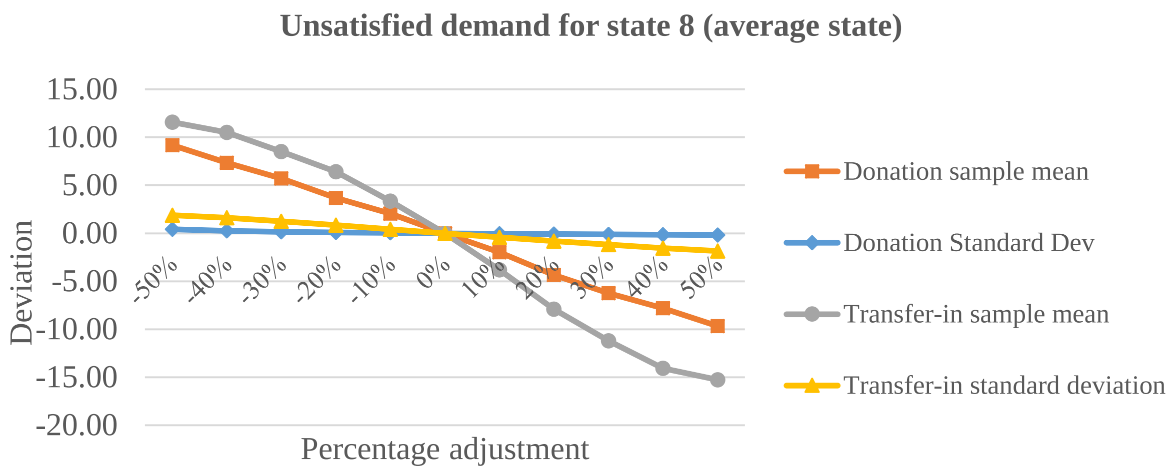

Figure 34. Deviation of unsatisfied demand for state 8.

From Figure 34, -50% to 0% percentage adjustments showed positive deviations from the base for all the 4 scenarios investigated. Positive deviations indicate higher unsatisfied demand compared to the base. As the supply percentage adjustments increased from 0% to 50% the deviations are negative. Negative deviations indicate lower unsatisfied demand compared to the base.

Figure 35 shows the unsatisfied demand for state 16 at the end of the time horizon for all the supply cases compared to that of the base scenario.

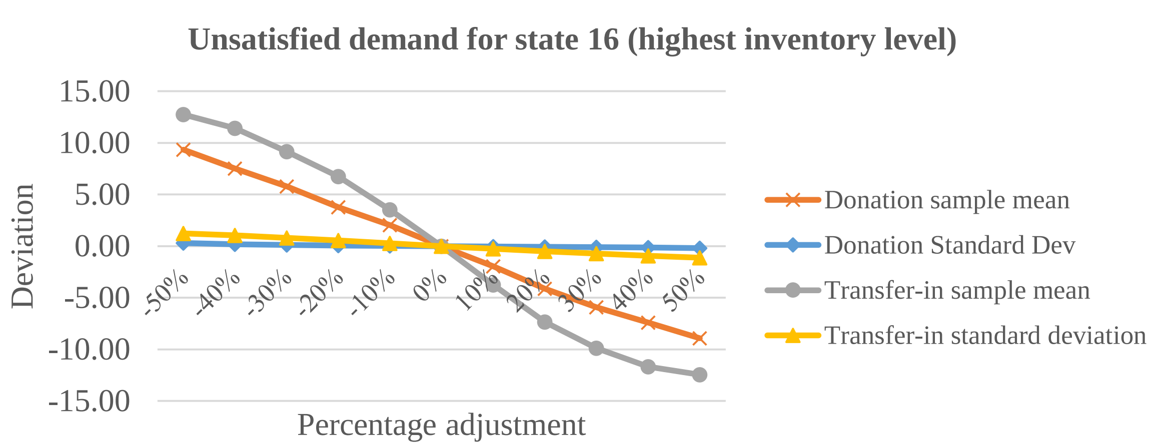

Figure 35. Deviation of unsatisfied demand for state 8.

From Figure 35, -50 to 0% percentage adjustments showed positive deviations from the base for all the 4 scenarios investigated. Positive deviations indicate higher unsatisfied demand compared to the base. As the demand percentage adjustments increased from 0% to 50% the deviations are negative. Once again, negative deviations indicate lower unsatisfied demand compared to the base.

In general, it can be observed that the deviation in the unsatisfied demand at the end of the time horizon from the base for each state investigated decreases monotonically as the percentage adjustment of the all the 4 variables for the supply increases.

5.3.2.3 Number of underserved counties for supply cases

Figures 36, 37 and 38, shows the underserved counties for state 1, 8 and 16 at the end of the time horizon for all the supply scenarios compared to that of the base scenario.

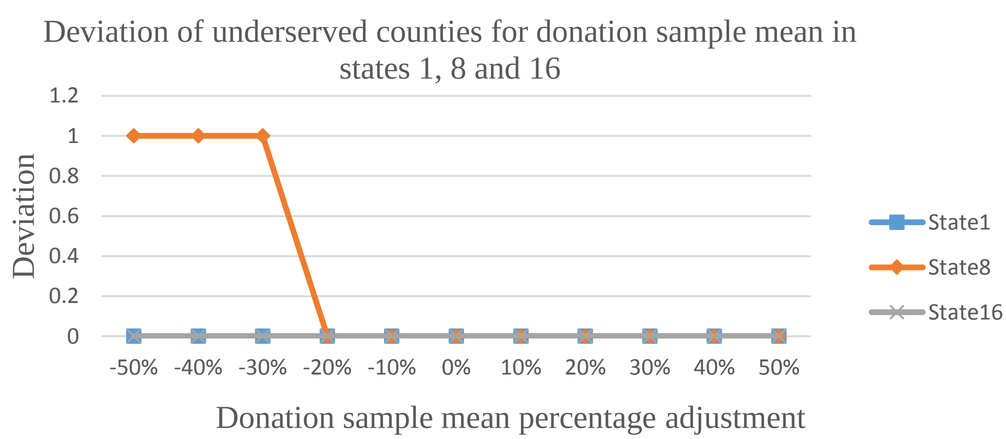

Figure 36. Deviation of underserved counties for donation sample mean.

From Figure 36, in state 8, when the donation sample mean percentage adjustment is from -50%, to -30%, the expected number of counties underserved from the base is 1. However, when the donation sample mean increases from -20% to 50%, the deviation from the base is zero, thus no county will be underserved in state 8. States 1 and 16 shows no deviation from the base.

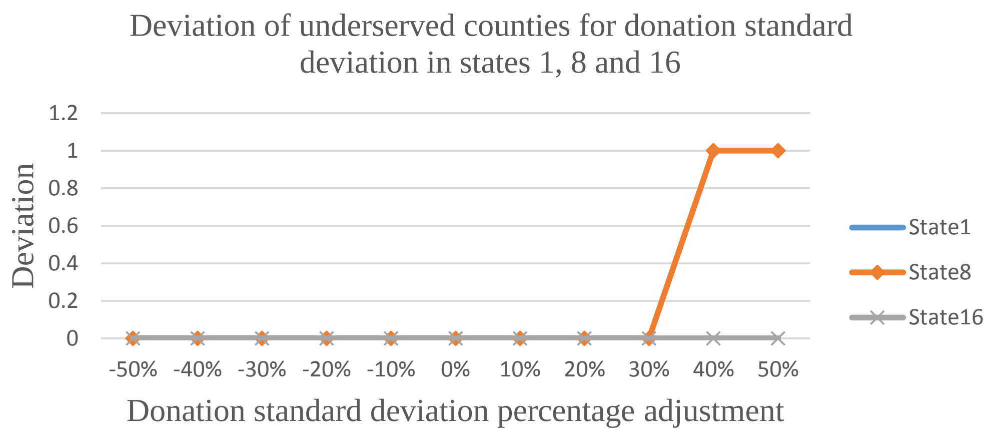

Figure 37. Deviation of underserved counties for donation standard deviation.

From Figure 37, in state 8, when the donation standard deviation increases from -50% to 30% the deviation from the base is zero. Thus, no county is underserved in state 8. However, when the donation standard deviation varies from 40% to 50%, the deviation from the base is 1. This implies that when the donation standard deviation increases from 40% to 50% above the actual value, then 1 county is underserved in state 8. Again, states 1 and 16 show no deviation from the base.

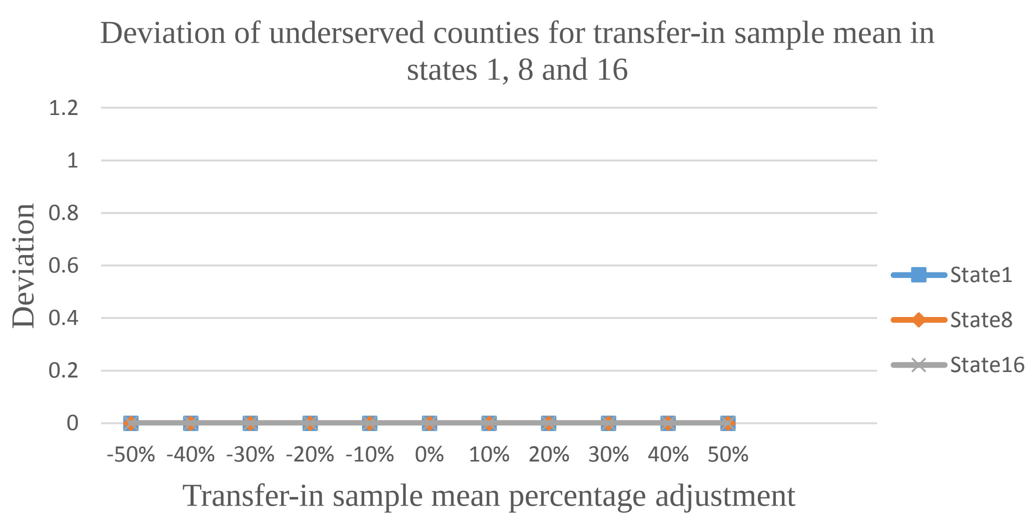

Figure 38. Deviation of underserved counties for transfer-in sample mean.

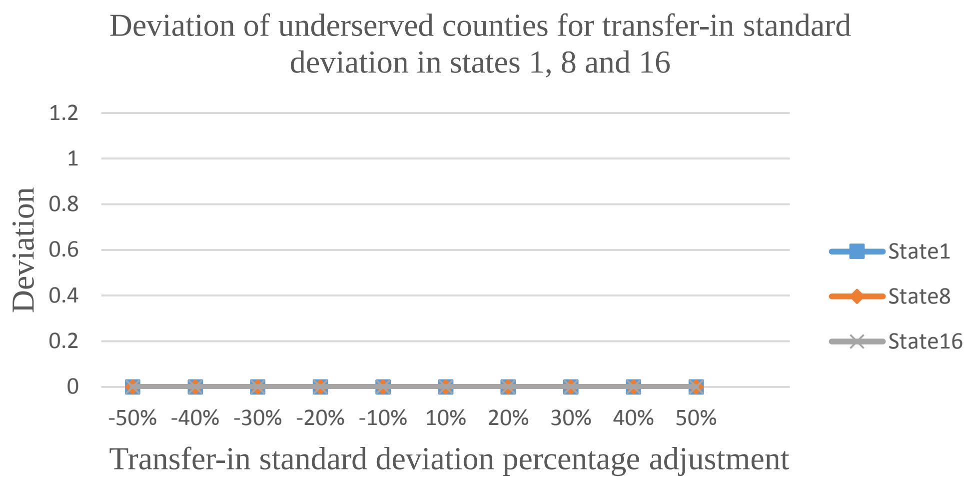

Figure 39. Deviation of underserved counties for transfer-in standard deviation.

From Figures 38 and 39, states 1, 8 and 16 show no deviation from the base. From the results of the base scenario the number of counties underserved in state 1 is six, state 8 is zero and in state 16 its zero. This implies that the changes in the transfer-in sample mean and standard deviation does not affect the number of counties underserved in states 1, 8 or 16. This is logical as the transfer-in comes after the distributions to the various counties.

5.3.3 Effects of non-stationary demand

5.3.3.1 Optimal policies for non-stationary demand

This experiment examines the situation where the county demands are non-stationary over the 12-month period. Each county demand assumes the projected demand and remains unchanged for the first six months and then fluctuates for the next six months by -50% to 100% in an increment of 10%. Figure 40 shows how the optimal policy structure changes as a result of the variations in the county demands. Once again, for the optimal policy structure only the constrained states are investigated.

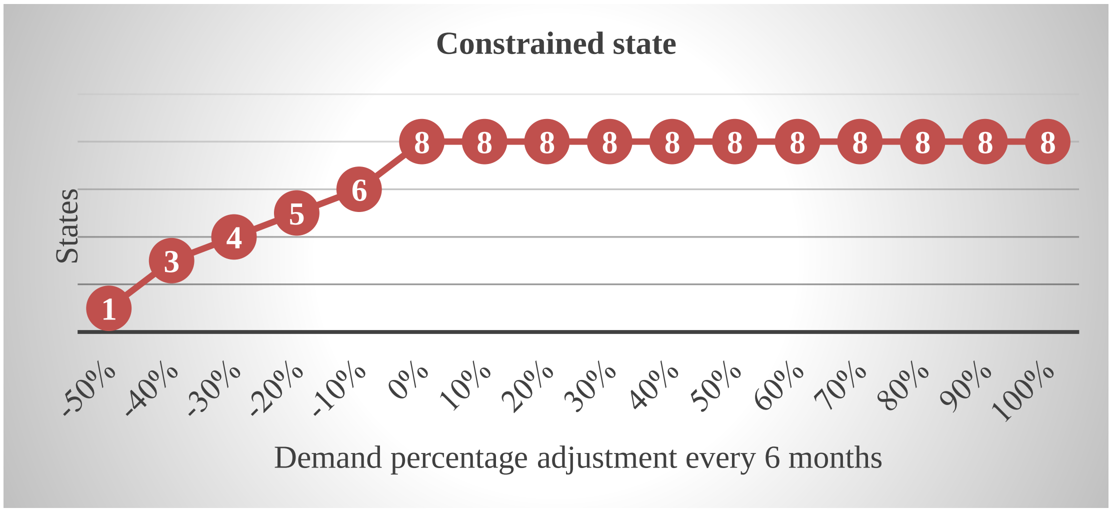

Figure 40. Optimal policy structure for non-stationary demand.

Figure 40, is interpreted as follows: when the demand percentage change for the last 6 months increases from -50% to 0% above the projected demand, the constrained states also increases from 1 to 8. However, when the demand is 0% to 100% above the projected demand constrained states remain at 8. In general, it can be observed that as the county demand increases from -50% to 100% the number of constrained available states also increases from 1 to 8.

5.3.3.2 Unsatisfied demand for non-stationary demand

The result for the unsatisfied demand for the non-stationary demand cases is displayed in Figure 41.

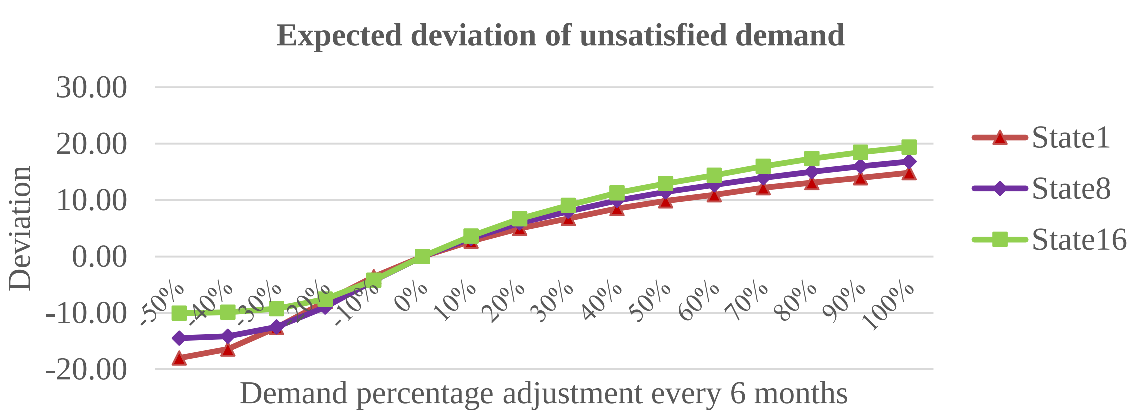

Figure 41. Unsatisfied demand for non-stationary demand.

From Figure 41, -50% to 0% demand percentage adjustment showed negative deviations from the base for all the states. Negative deviations indicate lower unsatisfied demand compared to that of the base. As the demand percentage adjustments increased from 0% to 50%, the deviations are positive. Positive deviations indicate higher unsatisfied demand compared to the base.

5.3.3.3 Number of counties underserved for non-stationary demand

Figure 42 shows the deviation from the base for the non-stationary demand cases**.**

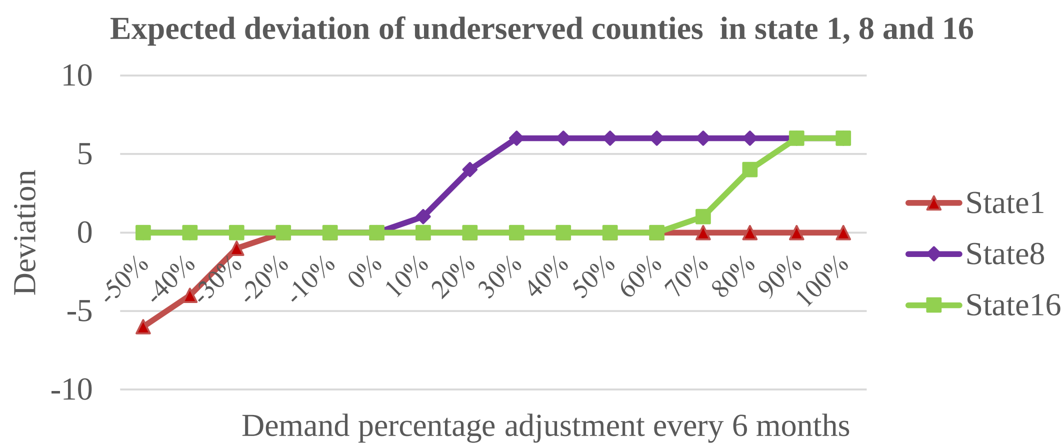

Figure 42. Relative percentage error of underserved counties.

From Figure 42, in state 1, -50% to -20% demand percentage adjustment showed negative deviations from the base. However, as the demand percentage adjustments increased from -20% to 100% the deviations are zero. Zero deviations indicate no change in the number of counties underserved compared to the base.

In state 8, -50 to 0% demand percentage adjustment showed no deviations from the base. However, as the demand percentage adjustments increased from 0% to 100% the deviations are positive. Positive deviations indicate bigger number of counties underserved compared to the base.

In state 16, -50 to 60% demand percentage adjustment showed no deviations from the base. However, as the demand percentage adjustments increased from 60% to 100% the deviations are positive. Positive deviations indicate higher number of counties underserved compared to the base.

In general, it can be observed that the deviation from the base for the number of counties underserved at the end of the time horizon increases monotonically as the demand percentage adjustment increases.

CHAPTER 6

Concluding Remarks and Future Work

6.1 Introduction

In 2012, the Economic Research Service (ERS) reported that approximately 15 % of households in the United States were food insecure. Food banks are non-profitable hunger-relief organizations that help in the fight against food insecurity by providing food and other services to people who are food insecure. Feeding America (FA) is the nation's leading domestic hunger-relief organization whose mission is to end hunger by feeding the hungry through a nationwide network of other food banks. The FA's benchmark is to distribute at least 75 pounds of products for each person in poverty over a 12-month period. A food bank is successful if its PPIP is 75 or more.

The Food bank of Central & Eastern North Carolina (FBCENC), a member of Feeding America network has six branches located in the Wilmington, Durham, Raleigh, Sandhills, Greenville, and New Bern areas in North Carolina. The FBCENC wants to use the performance indicator (PPIP) proposed by FA to measure the performance of its branches irrespective of the uncertainties in the supplies they receive. The main objective of this research is to develop a Discrete Time, Discrete Space Markov Decision Process to achieve the following objectives:

1. Find an optimal allocation policy that maximizes equity in the distribution of supplies using the PPIP;
2. Estimate the amount of unsatisfied demand for the counties whose PPIP are below 75;
3. Estimate the number of counties whose PPIPs may fall below the 75 target.

In this research we investigated 4 different allocations rules namely;

1. Serve the Largest Demand First: - the decision maker serves the county with the largest demand and proceeds down the hierarchy to serve the next larger demand and eventually serves the least demand last based on what is left after previous distributions;
2. Serve the Smallest First: - the decision maker serves the county with the smallest demand and proceeds up the hierarchy to serve the next smaller demand and eventually serves the highest demand last based on what is left after previous distributions;
3. Proportional Allocation: - the decision maker distribute supplies to the counties such that each county receives supplies based on the ratio of their poverty population to the total poverty population;
4. Fixed allocation: - the decision maker distributes fixed amount of supplies to each county irrespective of their demand sizes.

We used Puterman's backward induction algorithm to find the optimal allocation policy that maximizes equity in the distribution of supplies using the PPIP. Three major experiments are conducted to see how the optimal policy changes. These includes, changes in demand; changes in supply and non-stationary demand.

6.2 Conclusion

From this research, we found that the optimal supply allocation policy that maximizes equity in the distribution of supplies to counties using the PPIP criterion in general is as follows:

1. Allocation rule 3 should be used if the available inventory falls by at most 15% below the average available inventory irrespective of the time period

2. Allocation rules, 1 or 2 could be used in addition to allocation rule 3 if the available inventory is at least 15% above the sample mean irrespective of the time period
3. Exception: Allocation rule 3 should be used throughout the time period if the total county demand exceeds the available inventory to ensure equity
4. Allocation rule 3 is dominant in all the optimal policy structures for each time period irrespective of the states for all the cases considered in the sensitivity analyses.
5. Allocation rule 4 is only optimal when the available inventory is very large such that when equal amounts of supplies are distributed to all the counties, even the county with the largest demand is satisfied.

6.3 Recommendation for Durham Branch

Based on these experiments, we make the following recommendations to the Durham branch to assist the Durham branch distribute its supplies equitably and also to be able to meet there 75 pounds per person in poverty criterion:

1. The Durham branch should ensure that their available inventory is 15 % above the mean available inventory (i.e. 480,470 lb.) all the time. This will ensure that the branch can distribute supplies to meet the target PPIP of 75 pounds over a 12-month period irrespective of the incoming donations.
2. For available inventory states that are highly constrained, supplies should be distributed such that each county receives a proportion of supplies based on their share of the total poverty population
3. When adopted, the model's input parameters (sample means and standard deviations) should be updated from time to time as when more data becomes available.

6.4 Future Work

Irrespective of the work that has been done in this thesis, there is still room for improvement. The list below provides possible improvements that could be made to the model discussed in this thesis for an added advantage:

1. This model could be extended to all other branches in the FBCENC network to study branch to branch variability
2. One may model the donation and transfer-in with other probability distributions and analyze the associated prediction errors
3. Additional parameter such as transfer-out can also be added and modeled
4. Continuous state Markov Decision Process can be investigated to avoid discretization and the errors that may be associated with it.
5. Perishable items can be considered since this research only investigated the case of dry goods.
6. The warehouse capacity constraints can also be investigated to see how that may affect the optimal policy.

The model discussed in this thesis has laid a strong foundation for equitable distribution of supplies under uncertainty. Any additional effort including those outlined above as possible improvements to the model described in this thesis will go a long way to help develop mathematical models to equitably distribute limited food supplies to individuals at risk of hunger and its consequences.

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Appendix A

Sample Matlab Script for Discrete Time, Discrete Space Markov Decision Process

```
% Name of function: DTDSMDP.m
% Description: This script computes the Transition and Reward Matrices for a
%set of four policies and determines the best policy at a given state
% Assumptions:
% 1) Gaussian probability distribution is assumed for donations and transfer-in, which
% cause a transition from one state to the next.
% 2) Transfer comes in after the current demand (distributions) are served
% Note: Outputs are written to excel files in same folder one saves this script
% Subroutines
% FindState(PercentileRanges, PercentageValue)
% NormCDFTransitionProb(XPercentile, muX, SigX, XPercentileRange)
% ComputeActualValue(PercentileRanges, PercentageValue, ActualMean)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function DTDSMDP()
ExperimentNumber = 1;
format long;
% Inputs
scenario = 0;
ChathamDemand_PercGrowth   = [0]; % no adjustment
DurhamDemand_PercGrowth    = [0];
```

```
GranvilleDemand_PercGrowth  = [0];

OrangeDemand_PercGrowth     = [0];

PersonDemand_PercGrowth      = [0];

VanceDemand_PercGrowth       = [0];

NumOfPercAdjustments = length(ChathamDemand_PercGrowth);


TargetCountyPPIP = 75; % 75 pounds of food per person over 12 months

TargetPPIPMonthPeriod = 12; % number of months over which the target PPIP is computed

CountyPIPsOrig = [8028,36504,5770, 16475, 5829, 10859];

% Projected monthly demands for  Chatham, Durham, Granville, Orange, Person, Vance

ProjectedMonthlyCountyDemandsOrig =

TargetCountyPPIP.*CountyPIPsOrig/TargetPPIPMonthPeriod;


AvailableSampleMean = 418000; % in pounds


TransferSampleMean = 289000; % in pounds

TransferSampleSigma  = 31.98;% in percentage

TransferSampleMu = -2.5; % in percentage

TransSigPerDelta = 0;  % adjustment

TransMeanPercDelta =0; % adjustment


DonationSampleMean = 129000; % in pounds

DonationSampleSigma = 35.3; % in percentage
```

```
DonationSampleMu = -4.515; % in percentage
DonMeanPercDelta = 0; % adjustment
DonSigPerDelta = 0;   % adjustment

NumberOfCounties = 6;
NumberOfDecisions = 4;
EquityPercentageError = [];
UniqueBestAlternatives = [];
LossPercentageError{NumberOfCounties} = [];
UnderservedCountyPercentageError{NumberOfDecisions} = [];
UnderservedStatePercentageError{NumberOfDecisions}  = [];

AllExp1Scenarios = zeros(1, NumberOfCounties + 1);
for D=1:NumOfPercAdjustments
   scenario =  scenario + 1;
   CDemandGrowth = ChathamDemand_PercGrowth(D);
   DDemandGrowth = DurhamDemand_PercGrowth(D);
   GDemandGrowth = GranvilleDemand_PercGrowth(D);
   ODemandGrowth = OrangeDemand_PercGrowth(D);
   PDemandGrowth = PersonDemand_PercGrowth(D);
   VDemandGrowth = VanceDemand_PercGrowth(D);

   CountyDemandPercentageGrowth = [CDemandGrowth, DDemandGrowth, GDemandGrowth,
```

```
ODemandGrowth, PDemandGrowth, VDemandGrowth];
    AllExp1Scenarios(scenario,:) = [scenario, CountyDemandPercentageGrowth];

    ActualMeanDonation = (1+DonMeanPercDelta/100)*DonationSampleMean;
    sigmaDonation  = (1+DonSigPerDelta/100)*DonationSampleSigma;
    muDonation = DonationSampleMu;

    DonationPercentiles = [-Inf, -70, -60, -50, -40, -30, -20, -10, 0, 10, 20, 30, 40, 50, 60, 70, 80,
90, Inf]; % Donation percentages relative to its mean

    ActualMeanAvailInv = AvailableSampleMean;
    AvailInvPercentiles = [-Inf, -50, -40, -30, -20, -10, 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, Inf]; %
Available inventory upperbound percentages relative to its mean

    ActualMeanTransfer = (1+TransMeanPercDelta/100)*TransferSampleMean;
    sigmaTransf = (1+TransSigPerDelta/100)*TransferSampleSigma;
    muTransf = TransferSampleMu;  % Gaussian distribution parameter in percentage
    TransferPercentiles = [-Inf, -80, -70, -60, -50, -40, -30, -20, -10, 0, 10, 20, 30, 40, 50, 60, 70,
80, 90, Inf];

    %Calculate the actual demands based on poverty population growth
% for Chatham, Durham, Granville, Orange, Person, Vance Counties
```

```
    CountyDemands = ceil(max(0, (1 +
CountyDemandPercentageGrowth./100).*ProjectedMonthlyCountyDemandsOrig));
    CountyPIPs = floor(CountyDemands*TargetPPIPMonthPeriod./TargetCountyPPIP);

% Project previous 11 months
    Previous11MonthsDistToCounties = CountyDemands.*(TargetPPIPMonthPeriod-1);


    MaxTimeHorizon = 12; % time horizon in months
    NumberOfStates = length(AvailInvPercentiles) - 1; % number of intervals
    NumberOfDonationStates = length(DonationPercentiles) - 1; % number of intervals
    NumberOfTransferStates = length(TransferPercentiles) -1;  % number of intervals
    NumberOfEventsCausingTransitions = 2;
    NumberOfCounties = length(CountyDemands);
    AllDecisions = [1 2 3 4]; %All policies put together so one can run the policy iteration
    CountyPIPToTotalPIPs = zeros(1, NumberOfCounties) + CountyPIPs./sum(CountyPIPs); %
used for policy #3
    fixedAllocations = zeros(1, NumberOfCounties) + 1/NumberOfCounties; % default
    % Sanity Check against policies 3 and 4 being the same
    if(length(find(CountyPIPToTotalPIPs-fixedAllocations==0))==NumberOfCounties)
        AllDecisions(end) = []; % delete policy 4 if it is the same as 3
    end

    NumberOfDecisions = length(AllDecisions);
```

```
    AllP{NumberOfDecisions} = [];
    Q{NumberOfDecisions} = [];
    AllDistributionSequence{NumberOfDecisions} = [];

    CountyPPIPsHistory{NumberOfDecisions} = [];
    UnderservedRecord{NumberOfDecisions} = [];
    ImmediateExpectedDeviationFromTargetPPIP{NumberOfDecisions} = [];
    TotalUnderservedForEachState{NumberOfDecisions} = [];

    for action =1:NumberOfDecisions
        Decision = AllDecisions(action);
        P = zeros(NumberOfStates,NumberOfStates); % transition matrix
        Q{action} = zeros(NumberOfStates, 1); % Initial Immediate Expected Reward Expected
        AllDistributionSequence{action} =
zeros(NumberOfStates*NumberOfDonationStates*NumberOfTransferStates,
NumberOfEventsCausingTransitions+2+NumberOfCounties);
        CountyPPIPsHistory{action} = zeros(NumberOfStates*NumberOfDonationStates,
NumberOfCounties + 5);
        UnderservedRecord{action} = zeros(NumberOfStates*NumberOfDonationStates,
NumberOfCounties + 2);
        ImmediateExpectedDeviationFromTargetPPIP{action} = zeros(NumberOfStates,
NumberOfCounties);
        seqCount = 0;
```

```
count = 0;
TotalUnderservedForEachState{Decision} = zeros(1, NumberOfStates);
for i=1:NumberOfStates
    CurrentState = i;
    TempDeviationFromTargetPPIP = zeros(1, NumberOfCounties);
    TempUnderservedStateTotal = 0;
    for x=1:NumberOfDonationStates

        ActualAvailInv = ComputeActualValue(AvailInvPercentiles, AvailInvPercentiles(i+1),
ActualMeanAvailInv);
        [ActualDonation, IntervalMedianDonPer] =
ComputeActualValue(DonationPercentiles, DonationPercentiles(x+1), ActualMeanDonation);
        CurrentAvailInv = ActualAvailInv + ActualDonation; % currently available

        % Distribute Goods To Counties
        TempCurrentAvailInv = CurrentAvailInv;
        switch Decision
            case 1 %Decision #1 Evaluation: Serve the neediest first
                TempCountyDemands = CountyDemands;
                QntyRecievedByCounty = zeros(1, NumberOfCounties);
                for k=1:NumberOfCounties
                    [MaxD,Winner] = max(TempCountyDemands); % determines the max
                    QntyRecievedByCounty(Winner)  = floor(min(MaxD,TempCurrentAvailInv));
```

```
            TempCountyDemands(Winner) = -Inf; % prevents the previous winner
            TempCurrentAvailInv = max(0,TempCurrentAvailInv-MaxD);%surplus
        end

    case 2 %Decision #2 Evaluation: Serve the smallest demand first
        TempCountyDemands = CountyDemands;
        QntyRecievedByCounty = zeros(1, NumberOfCounties);

        for k=1:NumberOfCounties
            [MinD,Winner] = min(TempCountyDemands);
            QntyRecievedByCounty(Winner)  = floor(min(MinD,TempCurrentAvailInv));
            TempCountyDemands(Winner) = Inf; % prevents it from winning again
            TempCurrentAvailInv = max(0,TempCurrentAvailInv-MinD);%surplus
        end

    case 3 %Decision #3 Evaluation: Distribute According to Counties' PIPs
        QntyRecievedByCounty = zeros(1, NumberOfCounties);
        QntyRecievedByCounty(:,:) =
floor(min(TempCurrentAvailInv.*CountyPIPToTotalPIPs, CountyDemands));

    case 4 %Decision #4 Evaluation: Distribute According to Fixed Allocations
        QntyRecievedByCounty = zeros(1, NumberOfCounties);
        QntyRecievedByCounty(1,:) =
```

```
floor(min(TempCurrentAvailInv.*fixedAllocations, CountyDemands));

        end
        % Done Distributions

        AvailInvAfterDistribution  =  CurrentAvailInv - sum( QntyRecievedByCounty);
        DonationProb = NormCDFTransitionProb(DonationPercentiles(x+1), muDonation,
sigmaDonation, DonationPercentiles);

        for tf=1:NumberOfTransferStates
          % Compute The Next State And Transition Probability
          [ActualCurrentTransfer, IntervalMedianTransfPer] =
ComputeActualValue(TransferPercentiles, TransferPercentiles(tf+1), ActualMeanTransfer);
          NextAvailableInv = AvailInvAfterDistribution + ActualCurrentTransfer;

          NextAvailInvPercentile = 100*(NextAvailableInv-
ActualMeanAvailInv)/ActualMeanAvailInv; % in percentage
          NextState = FindState(AvailInvPercentiles, NextAvailInvPercentile);
          TransferProb = NormCDFTransitionProb(TransferPercentiles(tf+1), muTransf,
sigmaTransf, TransferPercentiles);
          P(CurrentState,NextState) = P(CurrentState,NextState) +
DonationProb*TransferProb;
```

```
%Compute Immediate Expected Reward
CurrentCountyPPIPs = floor((QntyRecievedByCounty +
Previous11MonthsDistToCounties)./CountyPIPs);
Reward = 1 - sum(abs( (CurrentCountyPPIPs -
mean(CurrentCountyPPIPs))/mean(CurrentCountyPPIPs ))); % preferred
Q{Decision}(CurrentState) = Q{Decision}(CurrentState) +
Reward*DonationProb*TransferProb;
TempDeviationFromTargetPPIP = TempDeviationFromTargetPPIP +
(TargetCountyPPIP - min(CurrentCountyPPIPs,
TargetCountyPPIP))*DonationProb*TransferProb;

%Keep history of the distribution records
seqCount =  seqCount + 1;
AllDistributionSequence{action}(seqCount,:) = [CurrentState,
IntervalMedianDonPer,  IntervalMedianTransfPer,  NextState, QntyRecievedByCounty];
end

count = count + 1;
NumberOfCountiesUnderserved = length(find(CurrentCountyPPIPs <
TargetCountyPPIP));
NumberOfCountiesWellServed = length(find(CurrentCountyPPIPs >=
TargetCountyPPIP));
TempUnderserved = zeros(1, NumberOfCounties);
```

```
            TempUnderserved(CurrentCountyPPIPs < TargetCountyPPIP) = 1;
            TempUnderserved = TempUnderserved*DonationProb; % expected
            UnderservedRecord{action}(count,:) = [CurrentState,TempUnderserved,
NumberOfCountiesUnderserved];
            CountyPPIPsHistory{action}(count,:) = [CurrentState, IntervalMedianDonPer,
TargetCountyPPIP, CurrentCountyPPIPs, NumberOfCountiesWellServed,
NumberOfCountiesUnderserved];
            TempUnderservedStateTotal = TempUnderservedStateTotal +
DonationProb*NumberOfCountiesUnderserved; %expected
         end

         % Compute The Expected Deviations From TargetPPIP
         ImmediateExpectedDeviationFromTargetPPIP{Decision}(CurrentState, :) =
TempDeviationFromTargetPPIP;
      TotalUnderservedForEachState{Decision}(i) = round(TempUnderservedStateTotal);
      end

      AllP{Decision} = P; % A structure containing all P matrices
   end

   %Policy Iteration.

   % Best Reward and Alternatives
```

```
BestReward = zeros(NumberOfStates, MaxTimeHorizon + 1);
BestAlternative = zeros(NumberOfStates, MaxTimeHorizon + 1);
OptimalEqn_Reward{MaxTimeHorizon + 1} = [];
OptimalEqn_Reward{MaxTimeHorizon+1} = zeros(NumberOfStates, NumberOfDecisions);
BestExpectedLoss{MaxTimeHorizon + 1} = [];

UniquePolicies = [];
for n = MaxTimeHorizon:-1:1
    BestExpectedLoss{MaxTimeHorizon+1} = zeros(NumberOfStates, NumberOfCounties);
    for action =1:NumberOfDecisions
        Decision = AllDecisions(action);
        OptimalEqn_Reward{n}(:,action) = AllP{Decision}*BestReward(:,n+1) + Q{Decision};
    end
    [BestReward(:,n), BestAlternative(:,n)] = max(OptimalEqn_Reward{n},[],2); % max along
the rows

    %Detect multiple decisions giving the same best reward. And also the best
    %expected loss associated withe the best policy
    LossPMatrix = zeros(NumberOfStates,NumberOfStates);
    ImmediateLossMatrix = zeros(NumberOfStates, NumberOfCounties);
    [ignore, PreviousStateDecision] = max(OptimalEqn_Reward{n}(1,:));
    for st=1:NumberOfStates
        TempOptimalEqn = OptimalEqn_Reward{n}(st,:);
```

```
maxReward = max(TempOptimalEqn); % max along the row
BestPolicies = find(TempOptimalEqn==maxReward);
UniquePolicies = unique([UniquePolicies, BestPolicies]);
strBestPolicy = '';
for bp=1:length(BestPolicies)
    strBestPolicy = strcat(strBestPolicy, num2str(BestPolicies(bp)));
end
BestAlternative(st,n) = str2num(strBestPolicy); % each digit represents a decision

%Construct a loss and immediate expected matrices associated with
%the best policy
StateDecisionPos = find( PreviousStateDecision==BestPolicies);
if(isempty(StateDecisionPos))
    StateDecision =  BestPolicies(1); % first in list
    PreviousStateDecision = StateDecision; % keep history
else
    StateDecision = PreviousStateDecision;
end
LossPMatrix(st, :)  = AllP{StateDecision}(st,:); % a row from the P matrix for the best reward at that state
ImmediateLossMatrix(st, :) = ImmediateExpectedDeviationFromTargetPPIP{StateDecision}(st,:); % a row from immediate loss
```

```
        %
      end
      BestExpectedLoss{n} = LossPMatrix*BestExpectedLoss{n+1} + ImmediateLossMatrix;
%% best loss
      % Done
  end



  % Compute equity error from the base
  format short;
  if scenario==1
      EquityPercentageError(scenario, :) = BestReward(:,1)'; % base model
  else
      EquityPercentageError(scenario, :) = (BestReward(:,1)' - EquityPercentageError(1, :));
  end



  %Find stable best alternative
  TransposeBA = BestAlternative';
  [ignore, rIndx] = unique(TransposeBA,'rows', 'last');
  timeHorizon = MaxTimeHorizon - rIndx(end) + 1;
  UniqueBestAlternatives(scenario, :) = [timeHorizon, fliplr(TransposeBA(rIndx(end),:))];
```

```
    % Compute the expected deviations from the base target PPIP
    format bank;
    if scenario==1
        for cnty=1:NumberOfCounties
            LossPercentageError{cnty}(scenario,:) = BestExpectedLoss{Jia,  #27}(:,cnty)'; % base
model at last time step
        end
    else
        for cnty=1:NumberOfCounties
            LossPercentageError{cnty}(scenario,:) = (BestExpectedLoss{Jia,  #27}(:,cnty)' -
LossPercentageError{cnty}(1,:));
        end
    end


    % Computeunderserved counts by county and by state for each action
    % relative to the base
    format short;
    Title = {'State', 'Lower Limit Don%', 'Upper Limit Don%',  'Chatham', 'Durham', 'Granville',
'Orange', 'Person', 'Vance',  '#Underserved'};
    for action=1:NumberOfDecisions
        DecisionName = action;
```

```
        if scenario==1
            UnderservedCountyPercentageError{DecisionName}(scenario,:) =
round(sum(UnderservedRecord{DecisionName})); % base model
            UnderservedStatePercentageError{DecisionName}(scenario,:) =
TotalUnderservedForEachState{DecisionName}; % base model
        else
            UnderservedCountyPercentageError{DecisionName}(scenario,:) =
(round(sum(UnderservedRecord{DecisionName}))-
UnderservedCountyPercentageError{DecisionName}(1,:));
            UnderservedStatePercentageError{DecisionName}(scenario,:) =
(TotalUnderservedForEachState{DecisionName}-
UnderservedStatePercentageError{DecisionName}(1,:));
        end
    end
end

%Save the summary results in excel
warning('off','MATLAB:xlswrite:AddSheet');
WorkBookName = strcat('Experiment', num2str(ExperimentNumber), 'Summary', '.xlsx');
WorkSheetName = 'Exp1Scenarious';
Title = {'Scenario','Chatham Demand Variation(%)','Durham Demand Variation(%)', 'Granville
Demand Variation(%)','Orange Demand Variation(%)', 'Person Demand Variation(%)', 'Vance
Demand Variation(%)'};
```

```
xlswrite(WorkBookName, Title, WorkSheetName, 'A1');
xlswrite(WorkBookName, AllExp1Scenarios, WorkSheetName, 'A2');

WorkSheetName = 'EquityError';
xlswrite(WorkBookName, EquityPercentageError, WorkSheetName, 'B2');

WorkSheetName = 'UniqueBestAlternative';
xlswrite(WorkBookName, UniqueBestAlternatives, WorkSheetName, 'B2');

Counties = {'Chatham', 'Durham', 'Granville', 'Orange', 'Person', 'Vance'};
for cnty=1:NumberOfCounties
   CountyName = char(Counties(cnty));
   WorkSheetName = strcat(CountyName, 'PercentageErrorLoss');
   xlswrite(WorkBookName, LossPercentageError{cnty}, WorkSheetName, 'B2')
end

Title = {'NumberOfState', 'Chatham', 'Durham', 'Granville', 'Orange', 'Person', 'Vance',
'#TotalUnderserved'};
for action=1:NumberOfDecisions
   DecisionName = action;

   WorkSheetName1 = strcat('Decision', num2str(DecisionName), 'UnderservedByCounty');
   xlswrite(WorkBookName, Title, WorkSheetName1, 'B1')
```

```
    xlswrite(WorkBookName,  UnderservedCountyPercentageError{DecisionName},
WorkSheetName1, 'B2');

    WorkSheetName2 = strcat('Decision', num2str(DecisionName), 'UnderservedByState');
    xlswrite(WorkBookName,  UnderservedStatePercentageError{DecisionName},
WorkSheetName2, 'B2');
end

warning('on','MATLAB:xlswrite:AddSheet');

end



%Name of subroutine: ComputeActualValue.m
%Description: This subroutine computes the actual value of a given
%percentage by using the median percentage of the range in which that given percentage falls.
%Inputs: PercentileRanges, PercentageValue
%Output: ActualValue
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [ActualValue, MedianPer] = ComputeActualValue(PercentileRanges, PercentageValue,
ActualMean)
if( isnumeric(PercentileRanges)  && isnumeric(PercentageValue) && isnumeric(ActualMean))
```

```
    if(PercentageValue<=PercentileRanges(2)) %finite lower bound
        MedianPer = PercentileRanges(2);
        ActualValue = max(0, (1 + PercentileRanges(2)/100)*ActualMean);
    elseif (PercentageValue>PercentileRanges(end-1)) % finite upper bound
        MedianPer = PercentileRanges(end-1);
        ActualValue = max(0, (1 + PercentileRanges(end-1)/100)*ActualMean);
    else
        PerPosition = find(PercentileRanges>=PercentageValue, 1, 'first'); % returns only the first
occurrence
        MedianPer = (PercentileRanges(PerPosition) + PercentileRanges(PerPosition-1))/2;
        ActualValue = max(0, (1 + MedianPer/100)*ActualMean);
    end
else
    ActualValue = -1; % error: cannot compute the ActualValue
end
end


%Name of subroutine: FindState.m
%Description: This subroutine finds the state a specific value belongs depending on which
interval it falls into
%Inputs: PercentileRanges, PercentageValue
%Output: State
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function State = FindState(PercentileRanges, PercentageValue)
if( isnumeric(PercentileRanges) && isnumeric(PercentageValue) )
    for rp=2:length(PercentileRanges)-1 % to take of Inf
        if PercentageValue<=PercentileRanges(rp)
            State = rp-1;
            break;
        elseif PercentageValue>PercentileRanges(end-1)
            State = length(PercentileRanges)-1; % greater than upper limit is also considered the last
interval
            break;
        end
    end
else
    State = -1; % error: cannot compute the state
end
end


%Name of subroutine: NormCDFTransitionProb.m
%Description: This subroutine calculates the probability of a given value falling
%between an interval a given interval using normal cumulative probability distribution
%Inputs: XPercentile, muX, SigX, XPercentileRange
%Output: px
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function px = NormCDFTransitionProb(XPercentile, muX, SigX, XPercentileRange)
if ( isnumeric(XPercentile) && isnumeric(muX) && isnumeric(SigX) &&
isnumeric(XPercentileRange) )
    if(XPercentile<=XPercentileRange(2))
        px = normcdf(XPercentileRange(2), muX, SigX); % Lower interval and downdards
    elseif(XPercentile>XPercentileRange(end-1))
        px = 1 - normcdf(XPercentileRange(end-1), muX, SigX); % Upper interval and upwards
    elseif(XPercentile>XPercentileRange(2) && XPercentile<=XPercentileRange(end-1));
        pos = FindState(XPercentileRange,XPercentile);
        px = normcdf(XPercentileRange(pos+1), muX, SigX) - normcdf(XPercentileRange(pos),
muX, SigX);
    end
else
    px = -1; % error: cannot compute the probability
end
end
```

Appendix B

Categories of records in the "FBCENC DataViewer" and their description

Database Record	Description
Distribution	Provide information about the amount of food items that the food bank branches distribute to agencies in the counties to give to aid recipients.
Donation	Provides information on all donations received by the food bank from Donor partners.
Transfer	Provides information on all transfer of supplies between the food bank branches. Transferring branch gives out items to the receiving branch.
Waste	Provides information on all food items that were lost as a result of food spoilage.
Custom Queries	These are standard queries that a user can select from a drop-down menu.

The key fields in the "FBCENC DataViewer" and their description

Key Fields	Description
Database	Contains all the fiscal years (FY). A fiscal year starts from July of one year to June of the next year.
Fiscal Month	The records in a fiscal year are aggregated by month to show all the transactions that occurred in a specific month of that fiscal year.
Branch Code	The food bank branch involved in a given transaction The branches are labelled by letters as follows: "D" for Durham, "G" for Greenville, "N" for New Bern, "R" for Raleigh, "S" Sandhills and "W" for Wilmington.
Product Type	This is used to identify the type of item involved in the transaction.
Storage Type	How the items are stored in the warehouses once received is one of the ways the food bank categories its supplies. The storage types include; dry, frozen, prepared, produce, refrigerate and salvage.

Appendix C

Detail experimental design and results of the sensitivity analysis

Experiment 1: Experimental design

Poverty population and demand percentage adjustment (%)						
CASES	Chatham	Durham	Granville	Orange	Person	Vance
-50%	-50	-50	-50	-50	-50	-50
-40%	-40	-40	-40	-40	-40	-40
-30%	-30	-30	-30	-30	-30	-30
-20%	-20	-20	-20	-20	-20	-20
-10%	-10	-10	-10	-10	-10	-10
0%	0	0	0	0	0	0
10%	10	10	10	10	10	10
20%	20	20	20	20	20	20
30%	30	30	30	30	30	30
40%	40	40	40	40	40	40
50%	50	50	50	50	50	50
60%	60	60	60	60	60	60
70%	70	70	70	70	70	70
80%	80	80	80	80	80	80
90%	90	90	90	90	90	90
100%	100	100	100	100	100	100

Experiment 1: Optimal stationary policy structure

Percentage	State16	State15	State14	State13	State12	State11	State10	State9	State8	State7	State6	State5	State4	State3	State2	State1
-50%	1234	1234	1234	1234	1234	123	123	123	123	123	123	123	123	123	123	3
-40%	1234	123	123	123	123	123	123	123	123	123	123	123	123	3	3	3
-30%	123	123	123	123	123	123	123	123	123	123	123	123	3	3	3	3
-20%	123	123	123	123	123	123	123	123	123	123	123	3	3	3	3	3
-10%	123	123	123	123	123	123	123	123	123	123	3	3	3	3	3	3
0%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
10%	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3	3
20%	123	123	123	123	123	123	3	3	3	3	3	3	3	3	3	3
30%	123	123	123	123	123	3	3	3	3	3	3	3	3	3	3	3
40%	123	123	123	3	3	3	3	3	3	3	3	3	3	3	3	3
50%	123	123	3	3	3	3	3	3	3	3	3	3	3	3	3	3
60%	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
70%	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
80%	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
90%	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
100%	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

Experiment 1: Deviation of unsatisfied demand (lb.) from the base scenario for each county

Percentages	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	-21.7	-21.6	-21.1	-20.6	-20.0	-19.4	-18.7	-18.1	-17.5	-16.9	-16.3	-15.6	-15.0	-14.4	-13.9	-13.6
-40%	-21.3	-21.3	-20.9	-20.6	-20.0	-19.4	-18.7	-18.1	-17.5	-16.9	-16.3	-15.6	-15.0	-14.4	-13.9	-13.6
-30%	-19.4	-19.7	-19.8	-19.7	-19.4	-19.0	-18.4	-17.9	-17.3	-16.8	-16.2	-15.6	-15.0	-14.4	-13.9	-13.6
-20%	-14.7	-14.9	-15.0	-15.4	-15.5	-15.6	-15.4	-15.3	-15.1	-14.9	-14.6	-14.3	-13.9	-13.5	-13.1	-12.8
-10%	-7.2	-7.2	-7.3	-7.5	-7.6	-7.8	-7.8	-7.8	-7.9	-8.0	-8.0	-8.0	-8.0	-8.0	-7.9	-7.8
0%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10%	5.4	5.4	5.5	5.5	5.6	5.8	5.9	5.9	5.9	6.0	6.1	6.2	6.2	6.3	6.3	6.3
20%	10.0	10.0	10.2	10.2	10.4	10.5	10.9	11.0	11.0	11.1	11.2	11.3	11.5	11.6	11.6	11.7
30%	13.6	13.6	13.8	13.8	14.1	14.3	14.6	14.9	15.0	15.1	15.2	15.3	15.5	15.7	15.9	15.9
40%	17.2	17.2	17.3	17.5	17.7	18.0	18.4	18.6	18.8	19.1	19.2	19.3	19.4	19.7	19.9	19.9
50%	19.9	19.9	20.1	20.2	20.6	20.7	21.1	21.5	21.7	21.9	22.3	22.4	22.5	22.7	22.9	23.0
60%	22.2	22.2	22.5	22.6	22.9	23.2	23.6	23.9	24.2	24.5	24.7	25.1	25.3	25.4	25.5	25.6
70%	24.6	24.6	24.9	25.1	25.3	25.6	26.2	26.4	26.7	27.0	27.4	27.6	28.0	28.3	28.4	28.4
80%	26.5	26.4	26.7	27.0	27.3	27.5	28.0	28.4	28.7	29.0	29.4	29.7	30.0	30.3	30.7	30.7
90%	28.3	28.4	28.6	28.8	29.2	29.4	29.9	30.3	30.7	30.9	31.3	31.7	32.1	32.3	32.6	32.8
100%	30.0	30.0	30.2	30.4	30.9	31.2	31.6	32.0	32.4	32.7	33.0	33.4	33.8	34.2	34.4	34.5

Experiment 1: Deviation of the number of counties underserved from the base scenario

	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	-6	-6	-6	-6	-5	-3	-1	0	0	0	0	0	0	0	0	0
-40%	-4	-5	-6	-6	-5	-3	-1	0	0	0	0	0	0	0	0	0
-30%	-1	-3	-5	-6	-5	-3	-1	0	0	0	0	0	0	0	0	0
-20%	0	0	-2	-4	-4	-3	-1	0	0	0	0	0	0	0	0	0
-10%	0	0	0	-1	-2	-2	-1	0	0	0	0	0	0	0	0	0
0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10%	0	0	0	0	1	2	2	1	0	0	0	0	0	0	0	0
20%	0	0	0	0	1	3	4	4	2	1	0	0	0	0	0	0
30%	0	0	0	0	1	3	5	6	5	3	1	0	0	0	0	0
40%	0	0	0	0	1	3	5	6	6	5	3	1	0	0	0	0
50%	0	0	0	0	1	3	5	6	6	6	5	3	1	0	0	0
60%	0	0	0	0	1	3	5	6	6	6	6	5	4	2	1	0
70%	0	0	0	0	1	3	5	6	6	6	6	6	6	5	3	1
80%	0	0	0	0	1	3	5	6	6	6	6	6	6	6	5	4
90%	0	0	0	0	1	3	5	6	6	6	6	6	6	6	6	6
100%	0	0	0	0	1	3	5	6	6	6	6	6	6	6	6	6

Experiment 2: Experimental design

Scenario 1

	Percentage adjustment (%)			
CASES	**Donation sample mean**	**Donation standard deviation**	**Transfer-in sample mean**	**Transfer-in standard deviation**
-50%	-50	0	0	0
-40%	-40	0	0	0
-30%	-30	0	0	0
-20%	-20	0	0	0
-10%	-10	0	0	0
0%	0	0	0	0
10%	10	0	0	0
20%	20	0	0	0
30%	30	0	0	0
40%	40	0	0	0
50%	50	0	0	0

Scenario 2

	Percentage adjustment (%)			
CASES	**Donation sample mean**	**Donation standard deviation**	**Transfer-in sample mean**	**Transfer-in standard deviation**
-50%	0	-50	0	0
-40%	0	-40	0	0
-30%	0	-30	0	0
-20%	0	-20	0	0
-10%	0	-10	0	0
0%	0	0	0	0
10%	0	10	0	0
20%	0	20	0	0
30%	0	30	0	0
40%	0	40	0	0
50%	0	50	0	0

Scenario 3

	Percentage adjustment (%)			
CASES	**Donation sample mean**	**Donation standard deviation**	**Transfer-in sample mean**	**Transfer-in standard deviation**
-50%	0	0	-50	0
-40%	0	0	-40	0
-30%	0	0	-30	0
-20%	0	0	-20	0
-10%	0	0	-10	0
0%	0	0	0	0
10%	0	0	10	0
20%	0	0	20	0
30%	0	0	30	0
40%	0	0	40	0
50%	0	0	50	0

Scenario 4

	Percentage adjustment (%)			
CASES	**Donation sample mean**	**Donation standard deviation**	**Transfer-in sample mean**	**Transfer-in standard deviation**
-50%	0	0	0	-50
-40%	0	0	0	-40
-30%	0	0	0	-30
-20%	0	0	0	-20
-10%	0	0	0	-10
0%	0	0	0	0
10%	0	0	0	10
20%	0	0	0	20
30%	0	0	0	30
40%	0	0	0	40
50%	0	0	0	50

Experiment 2: Optimal stationary policy structure

Donation sample mean																
Percentage a	State16	State15	State14	State13	State12	State11	State10	State9	State8	State7	State6	State5	State4	State3	State2	State1
-50%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-40%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-30%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-20%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-10%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
0%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
10%	123	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3
20%	123	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3
30%	123	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3
40%	123	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3
50%	123	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3

Donation standard deviation																
Percentage a	State16	State15	State14	State13	State12	State11	State10	State9	State8	State7	State6	State5	State4	State3	State2	State1
-50%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-40%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-30%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-20%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-10%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
0%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
10%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
20%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
30%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
40%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
50%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3

Transfer sample mean																
Percentage a	State16	State15	State14	State13	State12	State11	State10	State9	State8	State7	State6	State5	State4	State3	State2	State1
-50%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-40%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-30%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-20%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-10%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
0%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
10%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
20%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
30%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
40%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
50%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3

Transfer standard deviation																
Percentage a	State16	State15	State14	State13	State12	State11	State10	State9	State8	State7	State6	State5	State4	State3	State2	State1
-50%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-40%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-30%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-20%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
-10%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
0%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
10%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
20%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
30%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
40%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
50%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3

Experiment 2: Deviation of the unsatisfied demand (lb.) from the base scenario for each county

Donation sample mean																
Percentage adjustment	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	9.0	8.9	9.1	8.9	9.1	9.0	9.4	9.2	9.2	9.2	9.3	9.3	9.4	9.4	9.4	9.3
-40%	7.2	7.1	7.3	7.2	7.3	7.3	7.6	7.4	7.4	7.4	7.5	7.5	7.5	7.5	7.5	7.5
-30%	5.6	5.6	5.6	5.6	5.7	5.8	5.8	5.7	5.7	5.8	5.8	5.8	5.8	5.8	5.8	5.8
-20%	3.6	3.6	3.6	3.6	3.7	3.7	3.8	3.7	3.7	3.7	3.8	3.8	3.8	3.8	3.8	3.8
-10%	2.0	2.0	2.0	2.0	2.0	2.1	2.1	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
0%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10%	-1.9	-1.9	-1.9	-2.0	-1.9	-2.0	-1.9	-2.0	-2.0	-2.0	-2.0	-2.0	-2.0	-2.0	-2.0	-2.0
20%	-4.2	-4.3	-4.2	-4.3	-4.3	-4.4	-4.3	-4.3	-4.3	-4.3	-4.3	-4.3	-4.3	-4.2	-4.2	-4.1
30%	-6.0	-6.2	-6.1	-6.3	-6.2	-6.4	-6.2	-6.2	-6.2	-6.2	-6.2	-6.2	-6.2	-6.1	-6.0	-5.9
40%	-7.5	-7.5	-7.6	-7.6	-7.7	-7.8	-7.7	-7.8	-7.8	-7.8	-7.8	-7.8	-7.7	-7.6	-7.5	-7.4
50%	-9.4	-9.5	-9.4	-9.6	-9.6	-9.8	-9.7	-9.7	-9.7	-9.6	-9.6	-9.5	-9.4	-9.2	-9.0	-8.9

Donation Standard Deviation																
Percentage adjustment	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	0.5	0.5	0.4	0.5	0.5	0.6	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.3	0.3	0.3
-40%	0.3	0.3	0.3	0.3	0.3	0.4	0.2	0.3	0.3	0.3	0.3	0.2	0.2	0.2	0.2	0.2
-30%	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1
-20%	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
-10%	0.0	0.0	0.0	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10%	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20%	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
30%	-0.1	-0.1	-0.1	-0.1	-0.2	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
40%	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.1	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.1
50%	-0.2	-0.2	-0.2	-0.2	-0.3	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2

Transfer-in sample mean																
Percentage adjustment	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	10.9	10.9	10.9	10.9	11.0	11.1	11.3	11.6	11.8	12.0	12.1	12.2	12.4	12.5	12.7	12.7
-40%	10.0	10.0	10.0	10.0	10.1	10.2	10.3	10.5	10.7	10.8	10.9	11.0	11.2	11.3	11.4	11.4
-30%	8.1	8.1	8.1	8.1	8.2	8.2	8.4	8.5	8.6	8.7	8.8	8.9	9.0	9.1	9.1	9.1
-20%	6.2	6.2	6.2	6.2	6.2	6.3	6.3	6.4	6.5	6.5	6.6	6.6	6.7	6.7	6.7	6.8
-10%	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.4	3.4	3.4	3.4	3.5	3.5	3.5	3.5	3.5
0%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10%	-3.7	-3.7	-3.7	-3.7	-3.7	-3.7	-3.8	-3.8	-3.8	-3.9	-3.9	-3.9	-3.9	-3.9	-3.8	-3.8
20%	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.9	-7.9	-7.9	-7.9	-7.9	-7.8	-7.7	-7.6	-7.5	-7.3
30%	-11.2	-11.2	-11.2	-11.2	-11.2	-11.2	-11.2	-11.2	-11.2	-11.1	-11.0	-10.8	-10.6	-10.4	-10.1	-9.9
40%	-14.3	-14.3	-14.3	-14.3	-14.3	-14.3	-14.2	-14.1	-13.9	-13.7	-13.4	-13.1	-12.7	-12.3	-11.9	-11.7
50%	-15.3	-15.3	-15.3	-15.4	-15.4	-15.4	-15.4	-15.3	-15.1	-14.8	-14.4	-14.0	-13.6	-13.2	-12.7	-12.4

Transfer-in standard deviation																
Percentage adjustment	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	2.0	2.0	2.0	2.0	2.0	2.0	2.0	1.9	1.8	1.7	1.7	1.6	1.5	1.4	1.3	1.2
-40%	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.6	1.5	1.5	1.4	1.4	1.3	1.2	1.1	1.0
-30%	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.1	1.1	1.0	1.0	0.9	0.9	0.8
-20%	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.8	0.7	0.7	0.7	0.6	0.6	0.5
-10%	0.5	0.5	0.5	0.5	0.5	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.3	0.3	0.3	0.3
0%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10%	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.3	-0.3	-0.3	-0.3	-0.3
20%	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.8	-0.8	-0.7	-0.7	-0.7	-0.6	-0.6	-0.5	-0.5
30%	-1.3	-1.3	-1.3	-1.3	-1.3	-1.3	-1.2	-1.2	-1.1	-1.1	-1.0	-1.0	-0.9	-0.8	-0.8	-0.7
40%	-1.7	-1.7	-1.7	-1.7	-1.7	-1.6	-1.6	-1.5	-1.5	-1.4	-1.3	-1.3	-1.2	-1.1	-1.0	-0.9
50%	-2.0	-2.0	-2.0	-2.0	-2.0	-2.0	-1.9	-1.9	-1.8	-1.7	-1.6	-1.5	-1.4	-1.3	-1.2	-1.1

Experiment 2: Deviation of the number of counties under served from the base

Donation sample mean																
Percentage adjustment	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	0	0	0	0	1	3	4	1	0	0	0	0	0	0	0	0
-40%	0	0	0	0	1	3	3	1	0	0	0	0	0	0	0	0
-30%	0	0	0	0	1	2	2	1	0	0	0	0	0	0	0	0
-20%	0	0	0	0	1	2	1	0	0	0	0	0	0	0	0	0
-10%	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20%	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
30%	0	0	0	-1	-2	-2	0	0	0	0	0	0	0	0	0	0
40%	0	0	-1	-1	-2	-2	0	0	0	0	0	0	0	0	0	0
50%	0	0	-1	-2	-2	-2	-1	0	0	0	0	0	0	0	0	0

Donation Standard Deviation																
Percentage adjustment	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	0	0	0	0	1	1	-1	0	0	0	0	0	0	0	0	0
-40%	0	0	0	0	1	1	-1	0	0	0	0	0	0	0	0	0
-30%	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0	0
-20%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-10%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40%	0	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0
50%	0	0	0	-1	-1	0	1	1	0	0	0	0	0	0	0	0

Transfer-in sample mean																
Percentage adjustment	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-40%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-30%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-20%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-10%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Transfer-in standard deviation																
Percentage adjustment	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-40%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-30%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-20%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-10%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Experiment 3: Experimental design

Poverty population and demand percentage adjustment (%)						
CASES	Chatham	Durham	Granville	Orange	Person	Vance
-50%	-50	-50	-50	-50	-50	-50
-40%	-40	-40	-40	-40	-40	-40
-30%	-30	-30	-30	-30	-30	-30
-20%	-20	-20	-20	-20	-20	-20
-10%	-10	-10	-10	-10	-10	-10
0%	0	0	0	0	0	0
10%	10	10	10	10	10	10
20%	20	20	20	20	20	20
30%	30	30	30	30	30	30
40%	40	40	40	40	40	40
50%	50	50	50	50	50	50
60%	60	60	60	60	60	60
70%	70	70	70	70	70	70
80%	80	80	80	80	80	80
90%	90	90	90	90	90	90
100%	100	100	100	100	100	100

Experiment 3: Optimal policy structure

Percentage	State16	State15	State14	State13	State12	State11	State10	State9	State8	State7	State6	State5	State4	State3	State2	State1
-50%	1234	1234	1234	1234	1234	123	123	123	123	123	123	123	123	123	123	3
-40%	1234	123	123	123	123	123	123	123	123	123	123	123	123	3	3	3
-30%	123	123	123	123	123	123	123	123	123	123	123	123	3	3	3	3
-20%	123	123	123	123	123	123	123	123	123	123	123	3	3	3	3	3
-10%	123	123	123	123	123	123	123	123	123	123	3	3	3	3	3	3
0%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
10%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
20%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
30%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
40%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
50%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
60%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
70%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
80%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
90%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3
100%	123	123	123	123	123	123	123	123	3	3	3	3	3	3	3	3

Experiment 3: Deviation of the unsatisfied demand (lb.) from the base scenario for each county

Percentage	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	-18.0	-17.9	-17.4	-17.0	-16.4	-15.9	-15.1	-14.5	-13.9	-13.3	-12.7	-12.1	-11.5	-10.9	-10.3	-10.0
-40%	-16.4	-16.6	-16.3	-16.1	-15.7	-15.3	-14.7	-14.2	-13.6	-13.1	-12.5	-11.9	-11.3	-10.7	-10.2	-9.9
-30%	-12.6	-12.9	-13.1	-13.3	-13.2	-13.1	-12.8	-12.5	-12.2	-11.9	-11.5	-11.0	-10.6	-10.1	-9.6	-9.3
-20%	-8.0	-8.2	-8.4	-8.7	-8.9	-9.0	-8.9	-9.0	-9.0	-8.9	-8.8	-8.6	-8.4	-8.1	-7.8	-7.6
-10%	-3.6	-3.7	-3.8	-3.9	-4.0	-4.2	-4.2	-4.3	-4.3	-4.4	-4.5	-4.5	-4.5	-4.4	-4.3	-4.2
0%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10%	2.7	2.6	2.7	2.8	2.9	3.1	3.2	3.2	3.2	3.3	3.4	3.5	3.5	3.6	3.6	3.6
20%	5.0	4.9	5.1	5.2	5.3	5.5	5.8	5.9	6.0	6.0	6.1	6.3	6.4	6.5	6.6	6.6
30%	6.7	6.7	6.9	7.0	7.2	7.4	7.8	8.0	8.2	8.3	8.3	8.4	8.7	8.9	9.0	9.1
40%	8.5	8.5	8.7	8.8	9.1	9.3	9.7	9.9	10.1	10.4	10.6	10.6	10.8	11.0	11.2	11.3
50%	9.8	9.9	10.0	10.2	10.5	10.7	11.1	11.4	11.6	11.9	12.2	12.4	12.5	12.6	12.8	12.9
60%	10.9	11.0	11.3	11.4	11.6	11.9	12.4	12.7	13.0	13.3	13.5	13.8	14.1	14.2	14.3	14.4
70%	12.2	12.2	12.5	12.6	12.9	13.2	13.7	13.9	14.2	14.6	15.0	15.1	15.5	15.8	16.0	16.0
80%	13.1	13.0	13.4	13.6	13.9	14.1	14.6	15.0	15.3	15.6	16.0	16.3	16.6	16.9	17.3	17.3
90%	14.0	14.0	14.3	14.5	14.9	15.1	15.6	16.0	16.4	16.6	16.9	17.4	17.7	18.0	18.3	18.5
100%	14.8	14.8	15.1	15.3	15.7	16.0	16.4	16.8	17.2	17.6	17.9	18.2	18.7	19.0	19.2	19.4

Experiment 3: Deviation of the number of counties under served from the base scenario

	State1	State2	State3	State4	State5	State6	State7	State8	State9	State10	State11	State12	State13	State14	State15	State16
-50%	-6	-6	-6	-6	-5	-3	-1	0	0	0	0	0	0	0	0	0
-40%	-4	-5	-6	-6	-5	-3	-1	0	0	0	0	0	0	0	0	0
-30%	-1	-3	-5	-6	-5	-3	-1	0	0	0	0	0	0	0	0	0
-20%	0	0	-2	-4	-4	-3	-1	0	0	0	0	0	0	0	0	0
-10%	0	0	0	-1	-2	-2	-1	0	0	0	0	0	0	0	0	0
0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10%	0	0	0	0	1	2	2	1	0	0	0	0	0	0	0	0
20%	0	0	0	0	1	3	4	4	2	1	0	0	0	0	0	0
30%	0	0	0	0	1	3	5	6	5	3	1	0	0	0	0	0
40%	0	0	0	0	1	3	5	6	6	5	3	1	0	0	0	0
50%	0	0	0	0	1	3	5	6	6	6	5	3	1	0	0	0
60%	0	0	0	0	1	3	5	6	6	6	6	5	4	2	1	0
70%	0	0	0	0	1	3	5	6	6	6	6	6	6	5	3	1
80%	0	0	0	0	1	3	5	6	6	6	6	6	6	6	5	4
90%	0	0	0	0	1	3	5	6	6	6	6	6	6	6	6	6
100%	0	0	0	0	1	3	5	6	6	6	6	6	6	6	6	6